

3 Meetkunde

Voorkennis Goniometrische verhoudingen

Bladzijde 98

$$\begin{aligned} 1 \quad \tan(\angle A) &= \frac{BC}{AB} = \frac{3}{5} \text{ geeft } \angle A \approx 31,0^\circ \\ \sin(\angle D) &= \frac{EF}{DE} = \frac{8}{11} \text{ geeft } \angle D \approx 46,7^\circ \\ \cos(\angle I) &= \frac{HI}{GI} = \frac{4}{10} \text{ geeft } \angle I \approx 66,4^\circ \\ \tan(\angle K) &= \frac{LM}{KL} = \frac{7}{10} \text{ geeft } \angle K \approx 35,0^\circ \\ \cos(\angle P) &= \frac{\frac{1}{2}PQ}{PR} = \frac{7,5}{10} \text{ geeft } \angle P \approx 41,4^\circ \end{aligned}$$

Bladzijde 99

$$\begin{aligned} 2 \quad \cos(\angle C) &= \frac{AC}{BC} \\ \cos(38^\circ) &= \frac{AC}{17} \\ AC &= 17 \cos(38^\circ) \approx 13,4 \\ \tan(\angle E) &= \frac{DF}{DE} \\ \tan(55^\circ) &= \frac{DF}{5} \\ DF &= 5 \tan(55^\circ) \approx 7,1 \\ \sin(\angle G) &= \frac{HI}{GH} \\ \sin(40^\circ) &= \frac{7}{GH} \\ GH &= \frac{7}{\sin(40^\circ)} \approx 10,9 \\ \sin(\angle MLN) &= \frac{MN}{LN} \\ \sin(60^\circ) &= \frac{MN}{17} \\ KL = MN &= 17 \sin(60^\circ) \approx 14,7 \\ PS &= QS = 3 \\ \tan(\angle P) &= \frac{RS}{PS} \\ \tan(75^\circ) &= \frac{RS}{3} \\ RS &= 3 \tan(75^\circ) \approx 11,2 \end{aligned}$$

$$\begin{aligned} 3 \quad \cos(\angle BAC) &= \frac{AB}{AC} & \sin(\angle BAC) &= \frac{BC}{AC} \\ \cos(35^\circ) &= \frac{AB}{10} & \sin(35^\circ) &= \frac{BC}{10} \\ AB &= 10 \cos(35^\circ) = 8,191... & BC &= 10 \sin(35^\circ) = 5,735... \\ \text{omtrek rechthoek} &= 2 \cdot 8,191... + 2 \cdot 5,735... \approx 27,85 \end{aligned}$$

- 4 CD is de bij zijde AB horende hoogte.

$$\tan(\angle A) = \frac{CD}{\frac{1}{2}AB}$$

$$\tan(65^\circ) = \frac{CD}{4}$$

$$CD = 4 \tan(65^\circ) = 8,578...$$

$$\text{opp } \triangle ABC = \frac{1}{2} \cdot 8 \cdot 8,578... \approx 34,31$$

- 5 RS is de bij zijde PQ horende hoogte.

$$\text{opp } \triangle PQR = 20$$

$$\text{opp } \triangle PQR = \frac{1}{2} \cdot PQ \cdot RS \quad \left. \vphantom{\text{opp } \triangle PQR} \right\} \frac{1}{2} \cdot PQ \cdot RS = 20$$

$$\frac{1}{2} \cdot 6 \cdot RS = 20$$

$$RS = 6\frac{2}{3}$$

$$\tan(\angle P) = \frac{RS}{\frac{1}{2}PQ} = \frac{6\frac{2}{3}}{3} \text{ geeft } \angle P = 65,77...^\circ \approx 65,8^\circ$$

$$\angle Q = \angle P \approx 65,8^\circ$$

$$\angle R = 180^\circ - \angle P - \angle Q = 180^\circ - 2 \cdot 65,77...^\circ \approx 48,5^\circ$$

3.1 Berekeningen in driehoeken

Bladzijde 100

- 1 a In $\triangle ABC$ is $\tan(\angle A) = \frac{BC}{AB}$.

$$\angle A = 30^\circ \text{ en } AB = 6, \text{ dus je krijgt } \tan(30^\circ) = \frac{BC}{6} \text{ en hieruit volgt}$$

$$BC = 6 \tan(30^\circ) = 3,4641... \approx 3,464.$$

- b $\tan(\angle BDC) = \frac{BC}{BD}$

$$\tan(40^\circ) = \frac{3,4641...}{BD}$$

$$BD = \frac{3,4641...}{\tan(40^\circ)} \approx 4,13$$

Bladzijde 101

- 2 In $\triangle ABC$ is $\tan(28^\circ) = \frac{4}{AB}$

$$AB = \frac{4}{\tan(28^\circ)} = 7,52...$$

$$BM = \frac{1}{2}AB = 3,76...$$

$$\text{In } \triangle BCM \text{ is } \tan(\angle BMC) = \frac{BC}{BM} = \frac{4}{3,76...}$$

$$\angle BMC \approx 46,8^\circ$$

- 3 In $\triangle BCD$ is $\sin(70^\circ) = \frac{BC}{5}$

$$BC = 5 \sin(70^\circ) = 4,69...$$

$$\text{In } \triangle ABC \text{ is } \sin(\angle BAC) = \frac{BC}{AC} = \frac{4,69...}{10}$$

$$\angle BAC \approx 28,0^\circ$$

$$\angle ADC = 180^\circ - \angle BDC = 180^\circ - 70^\circ = 110^\circ$$

$$\angle ACD = 180^\circ - \angle ADC - \angle DAC \approx 180^\circ - 110^\circ - 28,0^\circ = 42,0^\circ$$

4 In $\triangle ABC$ is $\sin(40^\circ) = \frac{BC}{10}$
 $BC = 10 \sin(40^\circ) = 6,42\dots$

$$BM = \frac{1}{2}BC = 3,21\dots$$

In $\triangle ABC$ is $\cos(40^\circ) = \frac{AB}{10}$
 $AB = 10 \cos(40^\circ) = 7,66\dots$
 In $\triangle ABM$ is $\tan(\angle BAM) = \frac{BM}{AB} = \frac{3,21\dots}{7,66\dots}$
 $\angle BAM \approx 22,8^\circ$

Bladzijde 102

5 Stel $BM = x$, dan is $AB = 2x$.

In $\triangle ABC$ is $\tan(40^\circ) = \frac{BC}{2x}$
 $BC = 2x \tan(40^\circ)$

In $\triangle BCM$ is $\tan(\angle BMC) = \frac{BC}{x}$
 $BC = x \tan(\angle BMC)$

Dus $x \tan(\angle BMC) = 2x \tan(40^\circ)$
 $\tan(\angle BMC) = 2 \tan(40^\circ)$
 $\angle BMC \approx 59,2^\circ$

6 $\angle EAD = 90^\circ - \angle EAB = 90^\circ - 70^\circ = 20^\circ$
 Stel $DE = x$, dan is $CE = px$.

In $\triangle ADE$ is $\tan(20^\circ) = \frac{x}{AD}$

$$AD = \frac{x}{\tan(20^\circ)}$$

$\angle EBC = 90^\circ - \angle EBA = 90^\circ - 50^\circ = 40^\circ$

In $\triangle BCE$ is $\tan(40^\circ) = \frac{px}{BC}$

$$BC = \frac{px}{\tan(40^\circ)}$$

$BC = AD$, dus $\frac{px}{\tan(40^\circ)} = \frac{x}{\tan(20^\circ)}$
 $\frac{p}{\tan(40^\circ)} = \frac{1}{\tan(20^\circ)}$
 $p = \frac{\tan(40^\circ)}{\tan(20^\circ)} \approx 2,31$

7 Stel $BN = x$, dan is $BM = 2x$ en $AB = 4x$.

In $\triangle BCN$ is $\tan(25^\circ) = \frac{BN}{BC} = \frac{x}{BC}$, dus $x = BC \tan(25^\circ)$.

In $\triangle ABC$ is $\tan(\angle ACB) = \frac{AB}{BC} = \frac{4x}{BC} = \frac{4BC \tan(25^\circ)}{BC} = 4 \tan(25^\circ)$.

Dus $\angle ACB = 61,80\dots^\circ$

In $\triangle BCM$ is $\tan(\angle MCB) = \frac{BM}{BC} = \frac{2x}{BC} = \frac{2BC \tan(25^\circ)}{BC} = 2 \tan(25^\circ)$.

Dus $\angle MCB = 43,00\dots^\circ$

$\angle ACM = \angle ACB - \angle MCB = 61,80^\circ - 43,00\dots^\circ \approx 18,8^\circ$

8 De driehoeken ABC en DEC zijn gelijkvormig.

Bladzijde 104

9 $\left. \begin{array}{l} \angle DAC = \angle BAC \\ \angle AED = \angle ABC \text{ (gegeven)} \end{array} \right\} \triangle ADE \sim \triangle ACB$

$$\frac{AD}{AC} \mid \frac{AE}{AB} \mid \frac{DE}{BC} \text{ geeft } \frac{3}{AC} \mid \frac{5}{9} \mid \frac{DE}{7}$$

$$DE = \frac{5 \cdot 7}{9} = 3\frac{8}{9}$$

$$AC = \frac{3 \cdot 9}{5} = 5\frac{2}{5}, \text{ dus } CE = AC - AE = 5\frac{2}{5} - 5 = \frac{2}{5}.$$

10 $\left. \begin{array}{l} \angle ADE = \angle AEB \text{ (rechte hoek)} \\ \angle DEA = \angle BAE \text{ (Z-hoeken)} \end{array} \right\} \triangle ADE \sim \triangle BEA$

De stelling van Pythagoras in $\triangle ABE$ geeft $BE = \sqrt{10^2 - 6^2} = 8$.

$$\frac{AD}{BE} \mid \frac{AE}{AB} \mid \frac{DE}{AE} \text{ geeft } \frac{AD}{8} \mid \frac{6}{10} \mid \frac{DE}{6}$$

$$AD = \frac{6 \cdot 8}{10} = 4\frac{4}{5} \text{ en } DE = \frac{6 \cdot 6}{10} = 3\frac{3}{5}.$$

Bladzijde 105

11 $\left. \begin{array}{l} \angle BAC = \angle DAE \\ \angle ABC = \angle ADE \text{ (rechte hoek)} \end{array} \right\} \triangle ABC \sim \triangle ADE$

$$\frac{AB}{AD} \mid \frac{BC}{DE} \text{ geeft } \frac{20}{8} \mid \frac{16}{DE}, \text{ dus } DE = \frac{8 \cdot 16}{20} = 6\frac{2}{5}.$$

De stelling van Pythagoras in $\triangle BCD$ geeft $CD = \sqrt{12^2 + 16^2} = 20$.

$$\left. \begin{array}{l} \angle EDC = \angle BCD \text{ (Z-hoeken)} \\ \angle DEB = \angle CBE \text{ (Z-hoeken)} \end{array} \right\} \triangle DES \sim \triangle CBS \text{ dus } \frac{DE}{BC} \mid \frac{DS}{CS}$$

Stel $DS = x$, dan is $CS = 20 - x$.

$$\begin{aligned} \text{Dit geeft } & \frac{6\frac{2}{5}}{16} \mid \frac{x}{20-x} \\ & 16x = 6\frac{2}{5}(20-x) \\ & 16x = 128 - 6\frac{2}{5}x \\ & 22\frac{2}{5}x = 128 \\ & x = 5\frac{5}{7} \end{aligned}$$

$$\text{Dus } DS = 5\frac{5}{7}.$$

12 Zie de figuur hiernaast met hoogtelijn CF .

$$\left. \begin{array}{l} \angle FBC = \angle EBA \\ \angle BFC = \angle BEA \text{ (rechte hoek)} \end{array} \right\} \triangle BCF \sim \triangle BAE$$

$$\frac{BC}{AB} \mid \frac{BF}{BE} \text{ geeft } \frac{15}{10} \mid \frac{5}{BE}$$

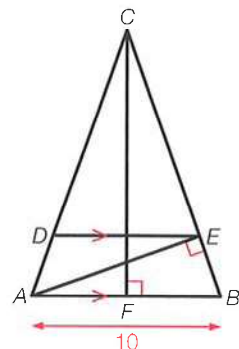
$$BE = \frac{5 \cdot 10}{15} = 3\frac{1}{3}$$

$$CE = BC - BE = 15 - 3\frac{1}{3} = 11\frac{2}{3}$$

$$\left. \begin{array}{l} \angle ACB = \angle DCE \\ \angle CAB = \angle CDE \text{ (F-hoeken)} \end{array} \right\} \triangle ABC \sim \triangle DEC$$

$$\frac{AB}{DE} \mid \frac{BC}{CE} \text{ geeft } \frac{10}{DE} \mid \frac{15}{11\frac{2}{3}}$$

$$DE = \frac{10 \cdot 11\frac{2}{3}}{15} = 7\frac{7}{9}$$



- 13 a** $\left. \begin{array}{l} \angle ABC = \angle DBE \\ \angle ACB = \angle DEB \text{ (F-hoeken)} \end{array} \right\} \triangle ABC \sim \triangle DBE \text{ dus } \frac{AB}{BD} = \frac{BC}{BE}$
 Stel $BD = x$, dan is $AB = x + 7$.

Dit geeft $\frac{x+7}{x} = \frac{10}{4}$

$$10x = 4(x+7)$$

$$10x = 4x + 28$$

$$6x = 28$$

$$x = 4\frac{2}{3}$$

Dus $BD = 4\frac{2}{3}$ en $AB = 11\frac{2}{3}$.

$$\left. \begin{array}{l} AC^2 + BC^2 = 6^2 + 10^2 = 136 \\ AB^2 = (11\frac{2}{3})^2 = 136\frac{1}{9} \end{array} \right\} AB^2 > AC^2 + BC^2, \text{ dus } \angle ACB > 90^\circ.$$

Dus bewering III is waar.

- b** Uit $\triangle ABC \sim \triangle DBE$ volgt $\frac{AC}{DE} = \frac{BC}{BE}$ oftewel $\frac{6}{DE} = \frac{10}{4}$.

Dit geeft $DE = \frac{4 \cdot 6}{10} = 2\frac{2}{5}$.

$$\left. \begin{array}{l} \angle CAE = \angle DEA \text{ (Z-hoeken)} \\ \angle ACD = \angle EDC \text{ (Z-hoeken)} \end{array} \right\} \triangle ACS \sim \triangle EDS$$

$$\frac{AC}{DE} = \frac{AS}{ES} \text{ geeft } \frac{6}{2\frac{2}{5}} = \frac{AS}{ES}$$

Hieruit volgt $AS : ES = 6 : 2\frac{2}{5} = 30 : 12 = 5 : 2$.

Dus AS is $\frac{5}{7}$ deel van AE .

- 14** Zie de figuur hiernaast met $MQ \perp CN$ en $MP = MN = 2$ (straal cirkel).

$$\left. \begin{array}{l} \angle DCN = \angle MNC \text{ (Z-hoeken)} \\ \angle CDN = \angle MQN \text{ (rechte hoek)} \end{array} \right\} \triangle CDN \sim \triangle NQM$$

De stelling van Pythagoras in $\triangle CDN$ geeft

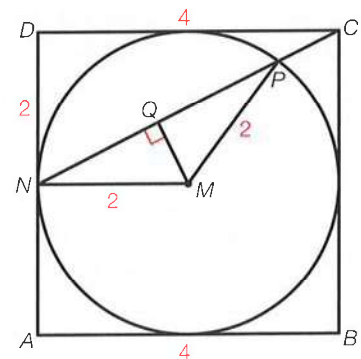
$$CN = \sqrt{4^2 + 2^2} = \sqrt{20} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}.$$

$$\frac{CN}{MN} = \frac{CD}{NQ} \text{ geeft } \frac{2\sqrt{5}}{2} = \frac{4}{NQ}$$

$$NQ = \frac{4 \cdot 2}{2\sqrt{5}} = \frac{4}{\sqrt{5}}$$

MQ is hoogtelijn van de gelijkbenige driehoek MNP , en dus is MQ symmetrieas van $\triangle MNP$.

$$\text{Dus } NP = 2NQ = 2 \cdot \frac{4}{\sqrt{5}} = \frac{8}{\sqrt{5}}.$$



Bladzijde 106

- 15 a** De som van de hoeken in een driehoek is 180° , dus in $\triangle ABC$ is $\angle A + \angle B + \angle C_{12} = 180^\circ$.

- b** In een gelijkbenige driehoek zijn de basishoeken gelijk.

$AM = CM$, dus $\triangle ACM$ is een gelijkbenige driehoek, dus $\angle A = \angle C_1$.

$BM = CM$, dus $\triangle BCM$ is een gelijkbenige driehoek, dus $\angle B = \angle C_2$.

$$\left. \begin{array}{l} \angle A + \angle B + \angle C_{12} = 180^\circ \\ \angle A = \angle C_1 \\ \angle B = \angle C_2 \end{array} \right\} \begin{array}{l} \angle C_1 + \angle C_2 + \angle C_{12} = 180^\circ \\ \angle C_{12} + \angle C_{12} = 180^\circ \\ 2 \cdot \angle C_{12} = 180^\circ \\ \angle C_{12} = 90^\circ \end{array}$$

Dus $\angle ACB = \angle C_{12} = 90^\circ$.

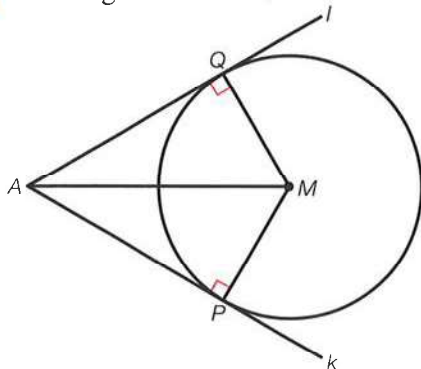
Bladzijde 108

- 16 a** $\left. \begin{array}{l} \angle ACB = \angle MNB \text{ (rechte hoek)} \\ \angle ABC = \angle MBN \end{array} \right\} \triangle ABC \sim \triangle MBN$
- b** $\frac{AB}{BM} \mid \frac{BC}{BN}$ geeft $\frac{2}{1} \mid \frac{BC}{BN}$, dus $BC = 2BN$,
 $\left. \begin{array}{l} BC = 2BN \\ BC = BN + CN \end{array} \right\} \begin{array}{l} 2BN = BN + CN \\ BN = CN \end{array}$
- c** De stelling van Pythagoras in $\triangle BMN$ geeft $BM^2 = BN^2 + MN^2$.
 Invullen van $BN = CN$ geeft $BM^2 = CN^2 + MN^2$.
 De stelling van Pythagoras in $\triangle CMN$ geeft $CM^2 = CN^2 + MN^2$.
 $BM^2 = CM^2$, dus $BM = CM$.
- d** $\left. \begin{array}{l} AM = BM \text{ (gegeven)} \\ BM = CM \text{ (zie c)} \end{array} \right\} A, B \text{ en } C \text{ liggen op de cirkel met middelpunt } M, \text{ straal } AM \text{ en middellijn } AB.$
 Hiermee is de stelling bewezen.

Bladzijde 109

- 17 a** De definitie zegt dat een raaklijn één punt met de cirkel gemeen heeft.
 Dus elk ander punt op de raaklijn ligt niet op de cirkel.
 Dit kan alleen als voor elk punt P van de raaklijn geldt $MP > MA$.
- b** Voor elk punt P waarvoor $P \neq A$ geldt dat $MP > MA$. Dus MA is de kortste verbinding van punt M tot raaklijn l . Uit de definitie van de afstand van een punt tot een lijn volgt dat $MA \perp l$.
- 18** $\left. \begin{array}{l} l \text{ raakt de cirkel met middelpunt } M \text{ in } A, \text{ dus } AM \perp l \\ l \text{ raakt de cirkel met middelpunt } N \text{ in } A, \text{ dus } AN \perp l \end{array} \right\} \angle MAN = 180^\circ, \text{ dus } MN \perp l.$

- 19** Zie de figuur hieronder.



$MP \perp k$ en $MQ \perp l$, dus $\angle APM = \angle AQM = 90^\circ$.

$$\left. \begin{array}{l} \text{In } \triangle APM \text{ is } AP^2 + MP^2 = AM^2. \\ \text{In } \triangle AQM \text{ is } AQ^2 + MQ^2 = AM^2. \end{array} \right\} \begin{array}{l} AP^2 + MP^2 = AQ^2 + MQ^2 \\ MP = MQ \text{ (straal cirkel)} \end{array} \left\} \begin{array}{l} AP^2 + MQ^2 = AQ^2 + MQ^2 \\ AP^2 = AQ^2 \\ AP = AQ \end{array}$$

- 20 a** $\angle ACB = 90^\circ$ (stelling van Thales)
 De stelling van Pythagoras in $\triangle ABC$ geeft $BC = \sqrt{6^2 - 3^2} = \sqrt{27} = \sqrt{9} \cdot \sqrt{3} = 3\sqrt{3}$.
 $\angle ADB = 90^\circ$ (stelling van Thales)
 De stelling van Pythagoras in $\triangle ABD$ geeft $BD = \sqrt{6^2 - 4^2} = \sqrt{20} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$.
- b** In $\triangle ABC$ is $\sin(\angle ABC) = \frac{AC}{AB} = \frac{3}{6}$, dus $\angle ABC = 30^\circ$.
 In $\triangle ABD$ is $\sin(\angle ABD) = \frac{AD}{AB} = \frac{4}{6}$, dus $\angle ABD = 41,81\dots^\circ$.
 $\angle CBD = \angle ABC + \angle ADC = 30^\circ + 41,81\dots^\circ = 71,81\dots^\circ \neq 90^\circ$, dus B ligt niet op de cirkel met middellijn CD (omgekeerde stelling van Thales).

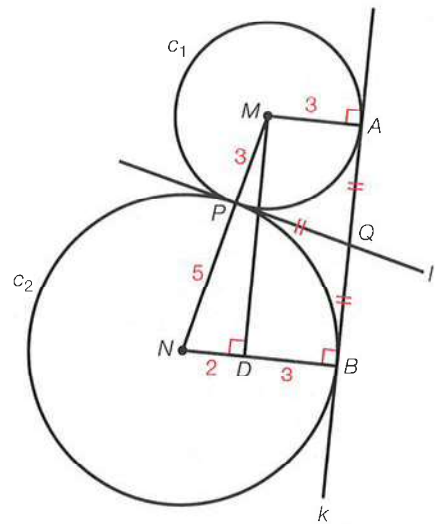
Bladzijde 110**21** Zie de figuur hiernaast. $PQ = AQ$ (stelling afstand punt tot raakpunten) $PQ = BQ$ (stelling afstand punt tot raakpunten)Dus ook $AQ = BQ$.Teken DM loodrecht op BN .De stelling van Pythagoras in $\triangle DMN$ geeft

$$DM = \sqrt{(5+3)^2 - 2^2} = \sqrt{60} = \sqrt{4 \cdot 15} = 2\sqrt{15}.$$

 $\angle MAB = 90^\circ$ (stelling raaklijn aan cirkel), $\angle NBA = 90^\circ$ (stelling raaklijn aan cirkel) en $\angle MDB = 90^\circ$,dus $ABDM$ is een rechthoek, dus $DM = AB$.

$$DM = AB = AQ + BQ = PQ + PQ = 2PQ,$$

$$\text{dus } PQ = \frac{1}{2} DM = \frac{1}{2} \cdot 2\sqrt{15} = \sqrt{15}.$$

**22** Zie de figuur hiernaast. $\angle ACB = 90^\circ$ (stelling van Thales) en $\angle BEA = 90^\circ$ (stelling van Thales) $\angle ACD = \angle BED$ (rechte hoek) $\angle ADC = \angle BDE$ (overstaande hoeken) $\} \triangle ACD \sim \triangle BED$

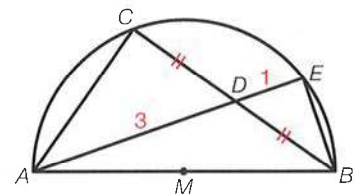
$$\frac{AC}{BE} \mid \frac{AD}{BD} \mid \frac{CD}{DE} \text{ geeft } \frac{AC}{BE} \mid \frac{3}{BD} \mid \frac{CD}{1}$$

$$BD = CD, \text{ dus } \frac{AC}{BE} \mid \frac{3}{BD} \mid \frac{BD}{1} \text{ en hieruit volgt } BD^2 = 3.$$

$$\text{Dus } BD = CD = \sqrt{3} \text{ en } BC = 2\sqrt{3}.$$

$$\text{De stelling van Pythagoras in } \triangle ACD \text{ geeft } AC = \sqrt{AD^2 - CD^2} = \sqrt{3^2 - (\sqrt{3})^2} = \sqrt{6}.$$

$$\text{In } \triangle ABC \text{ is } \tan(\angle BAC) = \frac{BC}{AC} = \frac{2\sqrt{3}}{\sqrt{6}} = \frac{\sqrt{4} \cdot \sqrt{3}}{\sqrt{6}} = \frac{\sqrt{12}}{\sqrt{6}} = \sqrt{2}.$$

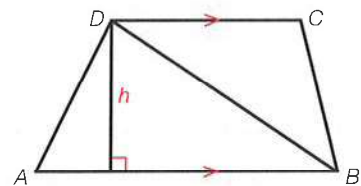
**3.2 Lengte, omtrek en oppervlakte****Bladzijde 112****23** Zie de figuur hiernaast met diagonaal BD .

$$O(\triangle ABD) = \frac{1}{2} \cdot AB \cdot h$$

$$O(\triangle BCD) = \frac{1}{2} \cdot CD \cdot h$$

$$O(ABCD) = O(\triangle ABD) + O(\triangle BCD)$$

$$O(ABCD) = \frac{1}{2} \cdot AB \cdot h + \frac{1}{2} \cdot CD \cdot h = \frac{1}{2}(AB + CD) \cdot h$$

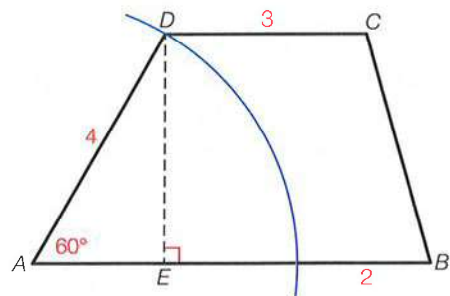
**Bladzijde 113****24** Zie de figuur hiernaast.

$$\text{In } \triangle ADE \text{ is } \sin(60^\circ) = \frac{DE}{4}, \text{ dus } DE = 4 \sin(60^\circ) = 3,46...$$

$$O(ABCD) = \frac{1}{2} (6 + 3) \cdot 3,46... = 15,58...$$

De oppervlakte van het gevraagde gebied is

$$15,58... - \frac{60}{360} \cdot \pi \cdot 4^2 \approx 7,21.$$



25 a Zie de figuur hiernaast.

$$AM = BM = CM = 5$$

$$\text{In } \triangle ADM \text{ is } DM = \sqrt{5^2 - 4^2} = 3.$$

$$O(\triangle ABC) = \frac{1}{2} \cdot 8 \cdot (5 + 3) = 32$$

De oppervlakte van het gevraagde gebied is

$$\pi \cdot 5^2 - 32 = 25\pi - 32.$$

b $O(\triangle ABM) = \frac{1}{2} \cdot 8 \cdot 3 = 12$

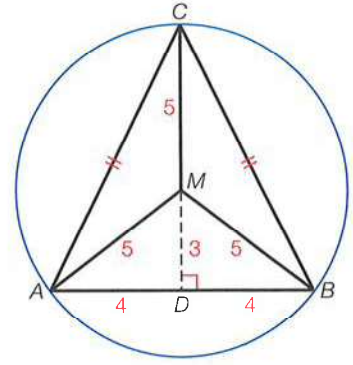
$$\text{In } \triangle ADM \text{ is } \sin(\angle AMD) = \frac{AD}{AM} = \frac{4}{5}$$

$$\angle AMD = 53,13\dots^\circ$$

$$\angle AMB = 2 \cdot \angle AMD = 2 \cdot 53,13\dots^\circ = 106,26\dots^\circ$$

$$\text{De gevraagde oppervlakte is } \frac{106,26\dots}{360} \cdot \pi \cdot 5^2 - 12 \approx 11,18.$$

$$\text{De gevraagde omtrek is } 8 + \frac{106,26\dots}{360} \cdot 2\pi \cdot 5 \approx 17,27.$$



26 $\angle ACB = 90^\circ$ (stelling van Thales), dus in $\triangle ABC$ is

$$BC = \sqrt{10^2 - 4^2} = \sqrt{84} = \sqrt{4} \cdot \sqrt{21} = 2\sqrt{21}.$$

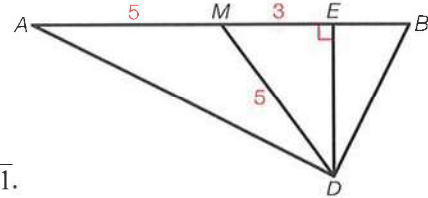
$$O(\text{blauwe gebied}) = O(\text{halve cirkel}) - O(\triangle ABC) = \frac{1}{2} \cdot \pi \cdot 5^2 - \frac{1}{2} \cdot 4 \cdot 2\sqrt{21} \\ = 12\frac{1}{2}\pi - 4\sqrt{21}$$

$$DM = 5 \text{ (straal cirkel), dus in } \triangle DEM \text{ is } DE = \sqrt{5^2 - 3^2} = 4.$$

$$O(\text{gele gebied}) = O(\text{halve cirkel}) - O(\triangle ABC) \\ = 12\frac{1}{2}\pi - \frac{1}{2} \cdot 10 \cdot 4 = 12\frac{1}{2}\pi - 20$$

Het gevraagde verschil is

$$12\frac{1}{2}\pi - 4\sqrt{21} - (12\frac{1}{2}\pi - 20) = 12\frac{1}{2}\pi - 4\sqrt{21} - 12\frac{1}{2}\pi + 20 = 20 - 4\sqrt{21}.$$

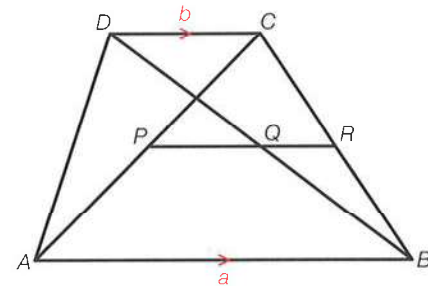


27 Zie de figuur hiernaast.

$$PR \text{ is een middenparallel van } \triangle ABC, \text{ dus } PR = \frac{1}{2}AB = \frac{1}{2}a.$$

$$QR \text{ is een middenparallel van } \triangle BCD, \text{ dus } QR = \frac{1}{2}CD = \frac{1}{2}b.$$

$$PQ = PR - QR = \frac{1}{2}a - \frac{1}{2}b = \frac{1}{2}(a - b)$$



28 Zie de figuur hiernaast.

$$\angle AMB = \frac{5}{12} \cdot 360^\circ = 150^\circ$$

$$\angle CMB = 180^\circ - \angle AMB = 180^\circ - 150^\circ = 30^\circ$$

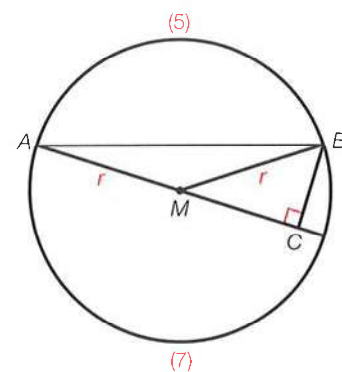
$$\text{In } \triangle BCM \text{ is } \sin(\angle BMC) = \frac{BC}{BM}$$

$$\sin(30^\circ) = \frac{BC}{r}$$

$$BC = r \sin(30^\circ) = \frac{1}{2}r$$

$$O(\triangle ABM) = \frac{1}{2} \cdot AM \cdot BC = \frac{1}{2} \cdot r \cdot \frac{1}{2}r = \frac{1}{4}r^2$$

$$\text{De oppervlakte van het kleinste gebied is } \frac{5}{12}\pi r^2 - \frac{1}{4}r^2.$$

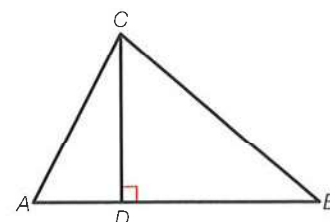


Bladzijde 114

29 Zie de figuur hiernaast.

$$\text{In } \triangle ACD \text{ is } \sin(\angle A) = \frac{CD}{AC}, \text{ dus } CD = AC \cdot \sin(\angle A).$$

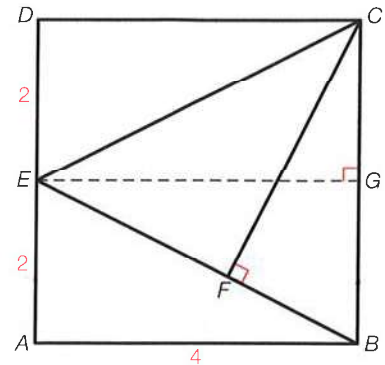
$$O(\triangle ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot AB \cdot AC \cdot \sin(\angle A)$$



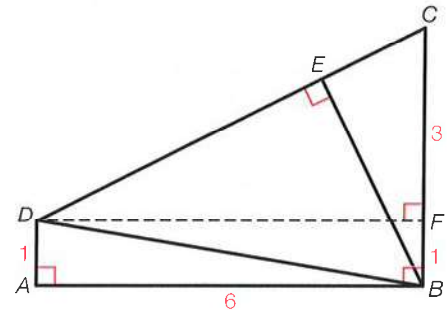
- 30 a $O(\triangle ABC) = \frac{1}{2} \cdot AB \cdot CE$
 b $O(\triangle ABC) = \frac{1}{2} \cdot BC \cdot AD$
 c Uit a en b volgt $\frac{1}{2} \cdot AB \cdot CE = \frac{1}{2} \cdot BC \cdot AD$.
 Vermenigvuldigen van het linker- en rechterlid met 2 geeft $AB \cdot CE = BC \cdot AD$.

Bladzijde 116

- 31 Zie de figuur hiernaast.
 $BE = \sqrt{4^2 + 2^2} = \sqrt{20} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$
 De zijde $\times$ hoogte-methode in $\triangle BCE$ geeft
 $BE \cdot CF = BC \cdot EG$
 $2\sqrt{5} \cdot CF = 4 \cdot 4$
 $CF = \frac{4 \cdot 4}{2\sqrt{5}} = \frac{16}{2\sqrt{5}} = \frac{8}{\sqrt{5}}$

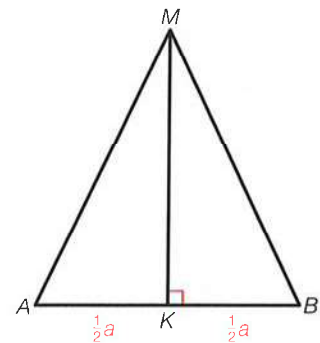


- 32 Zie de figuur hiernaast.
 $CD = \sqrt{6^2 + 3^2} = \sqrt{45} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$
 De zijde $\times$ hoogte-methode in $\triangle BCD$ geeft
 $CD \cdot BE = BC \cdot DF$
 $3\sqrt{5} \cdot BE = 4 \cdot 6$
 $BE = \frac{4 \cdot 6}{3\sqrt{5}} = \frac{24}{3\sqrt{5}} = \frac{8}{\sqrt{5}}$

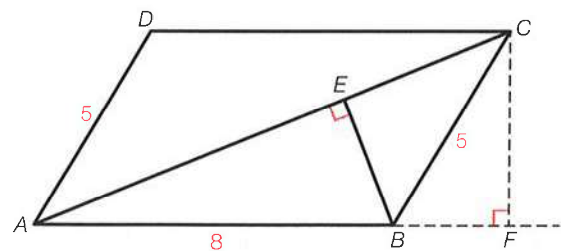


- 33 $\angle AMB = \frac{360^\circ}{7}$, dus $\angle AMK = \frac{360^\circ}{14}$.
 In $\triangle AKM$ is $\tan\left(\frac{360^\circ}{14}\right) = \frac{\frac{1}{2}a}{MK}$, dus $MK = \frac{\frac{1}{2}a}{\tan\left(\frac{360^\circ}{14}\right)} = 1,038...a$.

$$O(ABCDEFG) = 7 \cdot O(\triangle ABM) = 7 \cdot \frac{1}{2} \cdot a \cdot 1,038...a \approx 3,634a^2$$



- 34 Zie de figuur hiernaast.
 $O(ABCD) = AB \cdot CF = 8CF$
 $O(ABCD) = 32$ } $CF = 4$
 $BF = \sqrt{5^2 - 4^2} = 3$
 $AC = \sqrt{(8+3)^2 + 4^2} = \sqrt{137}$
 De zijde $\times$ hoogte-methode in $\triangle ABC$ geeft
 $AC \cdot BE = AB \cdot CF$
 $\sqrt{137} \cdot BE = 8 \cdot 4$
 $BE = \frac{8 \cdot 4}{\sqrt{137}} = \frac{32}{\sqrt{137}}$



35 Zie de figuur hiernaast.

$$O(\triangle ABN) = \frac{1}{2} \cdot r \cdot r \cdot \sin(80^\circ) = 0,4924...r^2$$

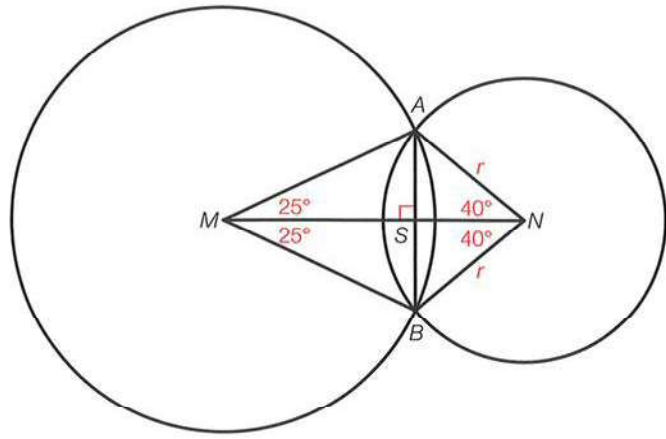
$$\text{In } \triangle ANS \text{ is } \sin(40^\circ) = \frac{AS}{r}, \text{ dus } AS = r \sin(40^\circ).$$

$$\text{In } \triangle AMS \text{ is } \sin(25^\circ) = \frac{AS}{AM} = \frac{r \sin(40^\circ)}{AM},$$

$$\text{dus } AM = \frac{r \sin(40^\circ)}{\sin(25^\circ)}.$$

$$O(\triangle ABM) = \frac{1}{2} \cdot \frac{r \sin(40^\circ)}{\sin(25^\circ)} \cdot \frac{r \sin(40^\circ)}{\sin(25^\circ)} \cdot \sin(50^\circ) \\ = 0,8860...r^2$$

$$O(\triangle AMBN) = O(\triangle ABN) + O(\triangle ABM) \\ = 0,4924...r^2 + 0,8860...r^2 \approx 1,378r^2$$



3.3 Rekenen met wortels

Bladzijde 118

36 II, III, VI

Bladzijde 120

37 a $2\sqrt{3} \cdot 3\sqrt{5} = 6\sqrt{15}$

b $\frac{5\sqrt{10}}{\sqrt{5}} = 5\sqrt{2}$

c $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1}{3}\sqrt{3}$

d $\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$

e $\sqrt{24} + \sqrt{6} = 2\sqrt{6} + \sqrt{6} = 3\sqrt{6}$

f $\sqrt{80} - \frac{10}{\sqrt{5}} = 4\sqrt{5} - \frac{10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = 4\sqrt{5} - \frac{10\sqrt{5}}{5} = 4\sqrt{5} - 2\sqrt{5} = 2\sqrt{5}$

38 a $\sqrt{4\frac{1}{2}} = \sqrt{\frac{9}{2}} = \frac{\sqrt{9}}{\sqrt{2}} = \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2} = 1\frac{1}{2}\sqrt{2}$

b $a\sqrt{8} - a\sqrt{2} = 2a\sqrt{2} - a\sqrt{2} = a\sqrt{2}$

c $(3\sqrt{2} - \sqrt{5})^2 = (3\sqrt{2})^2 - 2 \cdot 3\sqrt{2} \cdot \sqrt{5} + (\sqrt{5})^2 = 18 - 6\sqrt{10} + 5 = 23 - 6\sqrt{10}$

d $(2\sqrt{2} + 3\sqrt{3})^2 = (2\sqrt{2})^2 + 2 \cdot 2\sqrt{2} \cdot 3\sqrt{3} + (3\sqrt{3})^2 = 8 + 12\sqrt{6} + 27 = 35 + 12\sqrt{6}$

e $(a - a\sqrt{2})^2 = a^2 - 2a^2 \cdot \sqrt{2} + 2a^2 = 3a^2 - 2a^2\sqrt{2}$

f $(\frac{1}{2}\sqrt{5})^2 = \frac{1}{4} \cdot 5 = 1\frac{1}{4}$

39 a $3a\sqrt{2} \cdot a\sqrt{7} = 3a^2\sqrt{14}$

b $\frac{2\sqrt{14}}{3\sqrt{7}} = \frac{2}{3}\sqrt{2}$

c $\sqrt{12\frac{1}{2}} + \sqrt{24\frac{1}{2}} = \sqrt{\frac{25}{2}} + \sqrt{\frac{49}{2}} = \frac{\sqrt{25}}{\sqrt{2}} + \frac{\sqrt{49}}{\sqrt{2}} = \frac{5}{\sqrt{2}} + \frac{7}{\sqrt{2}} = \frac{12}{\sqrt{2}} = \frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$

d $(\frac{1}{2}\sqrt{2} + \frac{3}{4}\sqrt{3})^2 = (\frac{1}{2}\sqrt{2})^2 + 2 \cdot \frac{1}{2}\sqrt{2} \cdot \frac{3}{4}\sqrt{3} + (\frac{3}{4}\sqrt{3})^2 = \frac{1}{2} + \frac{3}{4}\sqrt{6} + \frac{27}{16} = 2\frac{3}{16} + \frac{3}{4}\sqrt{6}$

e $\sqrt{18a} - \sqrt{8a} = 3\sqrt{2a} - 2\sqrt{2a} = \sqrt{2a}$

f $\frac{1}{2}a\sqrt{2} \cdot \frac{1}{2}a\sqrt{3} = \frac{1}{4}a^2\sqrt{6}$

40 a $\sqrt{2a} + \sqrt{\frac{1}{2}a} = \sqrt{2a} + \sqrt{\frac{a}{2}} = \sqrt{2a} + \frac{\sqrt{a}}{\sqrt{2}} = \sqrt{2a} + \frac{\sqrt{a}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2a} + \frac{\sqrt{2a}}{2} = \sqrt{2a} + \frac{1}{2}\sqrt{2a} = 1\frac{1}{2}\sqrt{2a}$

b $(\frac{2}{3}a\sqrt{3})^2 + a^2\sqrt{7\frac{1}{9}} = (\frac{2}{3}a)^2 \cdot (\sqrt{3})^2 + a^2\sqrt{\frac{64}{9}} = \frac{4}{9}a^2 \cdot 3 + \frac{8}{3}a^2 = 1\frac{1}{3}a^2 + 2\frac{2}{3}a^2 = 4a^2$

c $(5\sqrt{3} + 2)(5\sqrt{3} - 2) = (5\sqrt{3})^2 - 2^2 = 75 - 4 = 71$

d $(a - \sqrt{3})^2 = a^2 - 2 \cdot a \cdot \sqrt{3} + (\sqrt{3})^2 = a^2 - 2a\sqrt{3} + 3$

e $(2 - \sqrt{2})^2 = 2^2 - 2 \cdot 2 \cdot \sqrt{2} + (\sqrt{2})^2 = 4 - 4\sqrt{2} + 2 = 6 - 4\sqrt{2}$

f $(1\frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{3})^2 = (1\frac{1}{2}\sqrt{2})^2 - 2 \cdot 1\frac{1}{2}\sqrt{2} \cdot \frac{1}{2}\sqrt{3} + (\frac{1}{2}\sqrt{3})^2 = 4\frac{1}{2} - 1\frac{1}{2}\sqrt{6} + \frac{3}{4} = 5\frac{1}{4} - 1\frac{1}{2}\sqrt{6}$

41 a $\sqrt{12a} + \sqrt{\frac{3}{4}a} = 2\sqrt{3a} + \frac{\sqrt{3a}}{\sqrt{4}} = 2\sqrt{3a} + \frac{\sqrt{3a}}{2} = 2\sqrt{3a} + \frac{1}{2}\sqrt{3a} = 2\frac{1}{2}\sqrt{3a}$

b $\sqrt{5\frac{1}{3}} - \sqrt{1\frac{1}{3}} = \sqrt{\frac{16}{3}} - \sqrt{\frac{4}{3}} = \frac{\sqrt{16}}{\sqrt{3}} - \frac{\sqrt{4}}{\sqrt{3}} = \frac{4}{\sqrt{3}} - \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3} = \frac{2}{3}\sqrt{3}$

c $\frac{9}{4\sqrt{2}} - \sqrt{2} = \frac{9}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} - \sqrt{2} = \frac{9\sqrt{2}}{8} - \sqrt{2} = 1\frac{1}{8}\sqrt{2} - \sqrt{2} = -\frac{7}{8}\sqrt{2}$

d $(\frac{1}{4}a\sqrt{2})^2 + (\frac{3}{4}a\sqrt{2})^2 = (\frac{1}{4}a)^2 \cdot (\sqrt{2})^2 + (\frac{3}{4}a)^2 \cdot (\sqrt{2})^2 = \frac{1}{16}a^2 \cdot 2 + \frac{9}{16}a^2 \cdot 2 = \frac{1}{8}a^2 + 1\frac{1}{8}a^2 = 1\frac{1}{4}a^2$

e $(2a\sqrt{2} - a\sqrt{3})^2 = (2a\sqrt{2})^2 - 2 \cdot 2a\sqrt{2} \cdot a\sqrt{3} + (a\sqrt{3})^2 = 8a^2 - 4a^2\sqrt{6} + 3a^2 = 11a^2 - 4a^2\sqrt{6}$

f $(\frac{1}{2\sqrt{2}} + \frac{3}{\sqrt{2}})^2 = (\frac{1}{2\sqrt{2}} + \frac{6}{2\sqrt{2}})^2 = (\frac{7}{2\sqrt{2}})^2 = \frac{49}{8} = 6\frac{1}{8}$

42 a $\frac{14}{3 - \sqrt{2}} = \frac{14}{3 - \sqrt{2}} \cdot \frac{3 + \sqrt{2}}{3 + \sqrt{2}} = \frac{14(3 + \sqrt{2})}{9 - 2} = 2(3 + \sqrt{2}) = 6 + 2\sqrt{2}$

b $\frac{4}{\sqrt{3} - 1} = \frac{4}{\sqrt{3} - 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{4(\sqrt{3} + 1)}{3 - 1} = 2(\sqrt{3} + 1) = 2\sqrt{3} + 2$

c $\frac{3\sqrt{2}}{\sqrt{2} + \sqrt{5}} = \frac{3\sqrt{2}}{\sqrt{2} + \sqrt{5}} \cdot \frac{\sqrt{2} - \sqrt{5}}{\sqrt{2} - \sqrt{5}} = \frac{3\sqrt{2}(\sqrt{2} - \sqrt{5})}{2 - 5} = -\sqrt{2}(\sqrt{2} - \sqrt{5}) = -2 + \sqrt{10}$

d $\frac{12\sqrt{2}}{\sqrt{10} - \sqrt{2}} = \frac{12\sqrt{2}}{\sqrt{10} - \sqrt{2}} \cdot \frac{\sqrt{10} + \sqrt{2}}{\sqrt{10} + \sqrt{2}} = \frac{12\sqrt{2}(\sqrt{10} + \sqrt{2})}{10 - 2}$
 $= 1\frac{1}{2}\sqrt{2}(\sqrt{10} + \sqrt{2}) = 1\frac{1}{2}\sqrt{20} + 1\frac{1}{2} \cdot 2 = 1\frac{1}{2} \cdot 2\sqrt{5} + 3 = 3\sqrt{5} + 3$

e $\frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} = \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} \cdot \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}} = \frac{(\sqrt{6} + \sqrt{2})^2}{6 - 2} = \frac{6 + 2\sqrt{12} + 2}{4} = \frac{8 + 4\sqrt{3}}{4} = 2 + \sqrt{3}$

f $\frac{6 - 5\sqrt{2}}{1 - \sqrt{2}} = \frac{6 - 5\sqrt{2}}{1 - \sqrt{2}} \cdot \frac{1 + \sqrt{2}}{1 + \sqrt{2}} = \frac{(6 - 5\sqrt{2})(1 + \sqrt{2})}{1 - 2} = \frac{6 + 6\sqrt{2} - 5\sqrt{2} - 10}{-1}$
 $= \frac{-4 + \sqrt{2}}{-1} = 4 - \sqrt{2}$

43 a $\sqrt{x} + \sqrt{y} = \sqrt{50}$ oftewel $\sqrt{x} + \sqrt{y} = 5\sqrt{2}$

Mogelijkheid 1

$$\sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$$

$$\sqrt{2} + \sqrt{16} \cdot \sqrt{2} = 5\sqrt{2}$$

$$\sqrt{2} + \sqrt{32} = 5\sqrt{2}$$

Hieruit volgt $x = 2$ en $y = 32$.

Mogelijkheid 2

$$2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$$

$$\sqrt{4} \cdot \sqrt{2} + \sqrt{9} \cdot \sqrt{2} = 5\sqrt{2}$$

$$\sqrt{8} + \sqrt{18} = 5\sqrt{2}$$

Hieruit volgt $x = 8$ en $y = 18$.

Dus $(x, y) = (2, 32)$ en $(x, y) = (8, 18)$ zijn oplossing van $\sqrt{x} + \sqrt{y} = \sqrt{50}$.

b Stel $(\sqrt{2} + 1)^7 = a$ en $(\sqrt{2} - 1)^7 = b$.

Zo krijg je $(a + b)^2 - (a - b)^2 = a^2 + 2ab + b^2 - (a^2 - 2ab + b^2) = 4ab$.

$4ab = 4(\sqrt{2} + 1)^7(\sqrt{2} - 1)^7 = 4((\sqrt{2} + 1)(\sqrt{2} - 1))^7 = 4(2 - 1)^7 = 4 \cdot 1^7 = 4$

44 a $2x = 16 - 8\sqrt{3}$
 $x = 8 - 4\sqrt{3}$

b $5x - 10\sqrt{2} = 50$
 $5x = 50 + 10\sqrt{2}$
 $x = 10 + 2\sqrt{2}$

Bladzijde 121

45 a $x\sqrt{5} + 2 = \sqrt{10}$

$x\sqrt{5} = -2 + \sqrt{10}$

$x = \frac{-2}{\sqrt{5}} + \frac{\sqrt{10}}{\sqrt{5}}$

$x = \frac{-2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} + \sqrt{2}$

$x = \frac{-2\sqrt{5}}{5} + \sqrt{2}$

$x = -\frac{2}{5}\sqrt{5} + \sqrt{2}$

b $7 - x\sqrt{3} = \sqrt{2}$

$x\sqrt{3} = 7 - \sqrt{2}$

$x = \frac{7}{\sqrt{3}} - \frac{\sqrt{2}}{\sqrt{3}}$

$x = \frac{7}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} - \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = \frac{7\sqrt{3}}{3} - \frac{\sqrt{6}}{3}$

$x = 2\frac{1}{3}\sqrt{3} - \frac{1}{3}\sqrt{6}$

c $\frac{1}{2}x\sqrt{2} + \sqrt{10} = 0$

$\frac{1}{2}x\sqrt{2} = -\sqrt{10}$

$x\sqrt{2} = -2\sqrt{10}$

$x = \frac{-2\sqrt{10}}{\sqrt{2}}$

$x = -2\sqrt{5}$

d $x\sqrt{8} + x\sqrt{2} = 12$

$x \cdot 2\sqrt{2} + x\sqrt{2} = 12$

$2x\sqrt{2} + x\sqrt{2} = 12$

$3x\sqrt{2} = 12$

$x = \frac{12}{3\sqrt{2}}$

$x = \frac{12}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$x = \frac{12\sqrt{2}}{6}$

$x = 2\sqrt{2}$

46 a $4x - 2x\sqrt{3} = \sqrt{6}$

$x(4 - 2\sqrt{3}) = \sqrt{6}$

$x = \frac{\sqrt{6}}{4 - 2\sqrt{3}}$

$x = \frac{\sqrt{6}}{4 - 2\sqrt{3}} \cdot \frac{4 + 2\sqrt{3}}{4 + 2\sqrt{3}}$

$x = \frac{\sqrt{6}(4 + 2\sqrt{3})}{16 - 12}$

$x = \frac{4\sqrt{6} + 2\sqrt{18}}{4}$

$x = \frac{4\sqrt{6} + 2 \cdot 3\sqrt{2}}{4}$

$x = \frac{4\sqrt{6} + 6\sqrt{2}}{4}$

$x = \sqrt{6} + 1\frac{1}{2}\sqrt{2}$

b $2x\sqrt{2} + x\sqrt{3} = 10$

$x(2\sqrt{2} + \sqrt{3}) = 10$

$x = \frac{10}{2\sqrt{2} + \sqrt{3}}$

$x = \frac{10}{2\sqrt{2} + \sqrt{3}} \cdot \frac{2\sqrt{2} - \sqrt{3}}{2\sqrt{2} - \sqrt{3}}$

$x = \frac{10(2\sqrt{2} - \sqrt{3})}{8 - 3}$

$x = 2(2\sqrt{2} - \sqrt{3})$

$x = 4\sqrt{2} - 2\sqrt{3}$

$$\text{c } x\sqrt{5} + x\sqrt{2} = \sqrt{10}$$

$$x(\sqrt{5} + \sqrt{2}) = \sqrt{10}$$

$$x = \frac{\sqrt{10}}{\sqrt{5} + \sqrt{2}}$$

$$x = \frac{\sqrt{10}}{\sqrt{5} + \sqrt{2}} \cdot \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}}$$

$$x = \frac{\sqrt{10}(\sqrt{5} - \sqrt{2})}{5 - 2}$$

$$x = \frac{\sqrt{50} - \sqrt{20}}{3}$$

$$x = \frac{5\sqrt{2} - 2\sqrt{5}}{3}$$

$$x = 1\frac{2}{3}\sqrt{2} - \frac{2}{3}\sqrt{5}$$

$$\text{d } x - \sqrt{2} = x\sqrt{3}$$

$$x - x\sqrt{3} = \sqrt{2}$$

$$x(1 - \sqrt{3}) = \sqrt{2}$$

$$x = \frac{\sqrt{2}}{1 - \sqrt{3}}$$

$$x = \frac{\sqrt{2}}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$$

$$x = \frac{\sqrt{2}(1 + \sqrt{3})}{1 - 3}$$

$$x = \frac{\sqrt{2} + \sqrt{6}}{-2}$$

$$x = -\frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{6}$$

$$\text{47 a } 2x\sqrt{6} - 6 = 4\sqrt{6}$$

$$2x\sqrt{6} = 6 + 4\sqrt{6}$$

$$x = \frac{6}{2\sqrt{6}} + \frac{4\sqrt{6}}{2\sqrt{6}}$$

$$x = \frac{6}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} + 2$$

$$x = \frac{6\sqrt{6}}{12} + 2$$

$$x = \frac{1}{2}\sqrt{6} + 2$$

$$\text{b } x\sqrt{3} + 6 = 2x$$

$$2x - x\sqrt{3} = 6$$

$$x(2 - \sqrt{3}) = 6$$

$$x = \frac{6}{2 - \sqrt{3}}$$

$$x = \frac{6}{2 - \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$x = \frac{6(2 + \sqrt{3})}{4 - 3}$$

$$x = 12 + 6\sqrt{3}$$

$$\text{c } 3\sqrt{2} + x\sqrt{2} = 4\sqrt{10}$$

$$x\sqrt{2} = 4\sqrt{10} - 3\sqrt{2}$$

$$x = \frac{4\sqrt{10}}{\sqrt{2}} - \frac{3\sqrt{2}}{\sqrt{2}}$$

$$x = 4\sqrt{5} - 3$$

$$\text{d } 3x\sqrt{2} - 2x = 6$$

$$x(3\sqrt{2} - 2) = 6$$

$$x = \frac{6}{3\sqrt{2} - 2}$$

$$x = \frac{6}{3\sqrt{2} - 2} \cdot \frac{3\sqrt{2} + 2}{3\sqrt{2} + 2}$$

$$x = \frac{6(3\sqrt{2} + 2)}{18 - 4}$$

$$x = \frac{18\sqrt{2} + 12}{14}$$

$$x = 1\frac{2}{7}\sqrt{2} + \frac{6}{7}$$

3.4 Vergelijkingen in de meetkunde

Bladzijde 123

$$\text{48 a } \text{In } \triangle ACD \text{ is } CD^2 = AC^2 - AD^2$$

$$CD^2 = (2a)^2 - a^2$$

$$CD^2 = 4a^2 - a^2$$

$$CD^2 = 3a^2$$

$$CD = \sqrt{3a^2} = \sqrt{a^2} \cdot \sqrt{3} = a\sqrt{3}$$

$$\text{b } \left. \begin{array}{l} CD = a\sqrt{3} \\ CD = 7\sqrt{3} \end{array} \right\} a\sqrt{3} = 7\sqrt{3}$$

$$a = 7$$

$$AC = 2a = 2 \cdot 7 = 14$$

Bladzijde 124

$$49 \quad AC = \frac{AB}{\sqrt{2}} = \frac{8}{\sqrt{2}} = \frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

$$DF = \frac{EF}{2} = \frac{6\sqrt{3}}{2} = 3\sqrt{3} \text{ en } DE = \sqrt{3} \cdot DF = \sqrt{3} \cdot 3\sqrt{3} = 3 \cdot 3 = 9$$

$$\left. \begin{aligned} MN &= \frac{KM}{2} = \frac{10}{2} = 5 \text{ en } KN = \sqrt{3} \cdot MN = 5\sqrt{3} \\ NL &= MN = 5 \end{aligned} \right\} KL = KN + NL = 5\sqrt{3} + 5$$

$$QS = RS \cdot \sqrt{2} = 3\sqrt{2} \text{ en } PQ = QS \cdot \sqrt{3} = 3\sqrt{2} \cdot \sqrt{3} = 3\sqrt{6}$$

- 50 Noem het middelpunt van de omgeschreven cirkel G.

$$\triangle ABG \text{ is gelijkbenig en } \angle AGB = \frac{360^\circ}{6} = 60^\circ,$$

$$\text{dus } \angle A = \angle B = \frac{180^\circ - 60^\circ}{2} = 60^\circ.$$

Hieruit volgt dat $\triangle ABG$ een gelijkzijdige driehoek is.

$$\text{Uit } AK = 4 \text{ en } \angle A = 60^\circ \text{ volgt } GK = \sqrt{3} \cdot AK = 4\sqrt{3}.$$

$$O(ABCDEF) = 6 \cdot O(\triangle ABG) = 6 \cdot \frac{1}{2} \cdot 8 \cdot 4\sqrt{3} = 96\sqrt{3}$$

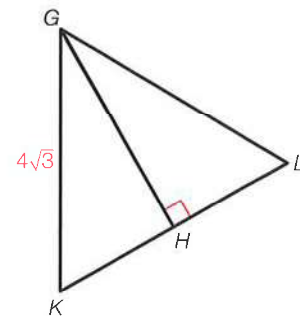
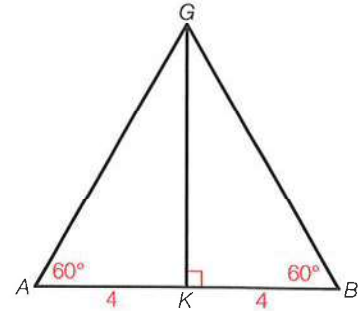
$\triangle GKL$ is een gelijkzijdige driehoek met zijde $4\sqrt{3}$, dus

$$KL = 4\sqrt{3}, KH = 2\sqrt{3} \text{ en } GH = \sqrt{3} \cdot KH = \sqrt{3} \cdot 2\sqrt{3} = 6.$$

$$O(KLMNOP) = 6 \cdot O(\triangle GKL) = 6 \cdot \frac{1}{2} \cdot 4\sqrt{3} \cdot 6 = 72\sqrt{3}$$

De oppervlakte van het gekleurde gebied is

$$O(ABCDEF) - O(KLMNOP) = 96\sqrt{3} - 72\sqrt{3} = 24\sqrt{3}.$$

**Bladzijde 125**

$$51 \quad \text{In } \triangle BCD \text{ is } BD = CD = \frac{BC}{\sqrt{2}} = \frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}.$$

$$\text{In } \triangle ACD \text{ is } AD = \sqrt{3} \cdot CD = \sqrt{3} \cdot 3\sqrt{2} = 3\sqrt{6}.$$

$$\begin{aligned} O(\triangle ABC) &= \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot (AD + BD) \cdot CD = \frac{1}{2} \cdot (3\sqrt{6} + 3\sqrt{2}) \cdot 3\sqrt{2} \\ &= 4\frac{1}{2}\sqrt{12} + 9 = 4\frac{1}{2} \cdot 2\sqrt{3} + 9 = 9\sqrt{3} + 9 \end{aligned}$$

$$52 \quad \text{a In } \triangle ABE \text{ is } AB = \sqrt{2} \text{ en } AE = 1.$$

$$\text{In } \triangle BDE \text{ is } DE = \sqrt{3} \text{ en } BD = 2.$$

$$AD = AE + DE = 1 + \sqrt{3}$$

$$\text{In } \triangle ACD \text{ is } AC = \frac{AD}{\sqrt{2}} = \frac{1 + \sqrt{3}}{\sqrt{2}} = \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2} = \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{6}.$$

$$CD = AC = \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{6}$$

$$BC = AC - AB = \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{6} - \sqrt{2} = \frac{1}{2}\sqrt{6} - \frac{1}{2}\sqrt{2}$$

$$\text{In } \triangle BCD \text{ is } \sin(15^\circ) = \frac{BC}{BD} = \frac{\frac{1}{2}\sqrt{6} - \frac{1}{2}\sqrt{2}}{2} = \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2}.$$

$$\text{b } \cos(15^\circ) = \frac{CD}{BD} = \frac{\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{6}}{2} = \frac{1}{4}\sqrt{2} + \frac{1}{4}\sqrt{6}$$

- 53 a** In $\triangle BCD$ is $CD = BD = x$.
 In $\triangle ACD$ is $AD = \sqrt{3} \cdot CD = \sqrt{3} \cdot x = x\sqrt{3}$.
- b** $AB = AD + BD = x\sqrt{3} + x$
 $AB = 10$ $\left. \vphantom{AB = 10} \right\} x\sqrt{3} + x = 10$

$$x\sqrt{3} + x = 10$$

$$x(\sqrt{3} + 1) = 10$$

$$x = \frac{10}{\sqrt{3} + 1}$$

$$x = \frac{10}{\sqrt{3} + 1} \cdot \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$x = \frac{10(\sqrt{3} - 1)}{3 - 1}$$

$$x = 5(\sqrt{3} - 1)$$

$$x = 5\sqrt{3} - 5$$

c $O(\triangle ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot 10 \cdot (5\sqrt{3} - 5) = 25\sqrt{3} - 25$

- 54** Zie de figuur hiernaast.

Stel $AD = x$.

In $\triangle ACD$ is $CD = \sqrt{3} \cdot AD = x\sqrt{3}$.

In $\triangle BCD$ is $BD = CD = x\sqrt{3}$.

$$AB = AD + BD = x + x\sqrt{3}$$

$$AB = 12 \quad \left. \vphantom{AB = 12} \right\} \begin{aligned} x + x\sqrt{3} &= 12 \\ x(1 + \sqrt{3}) &= 12 \end{aligned}$$

$$x = \frac{12}{1 + \sqrt{3}}$$

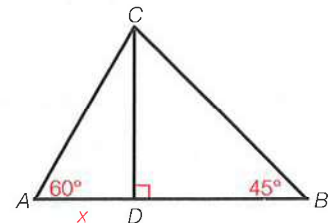
$$x = \frac{12}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$$

$$x = \frac{12(1 - \sqrt{3})}{1 - 3}$$

$$x = -6(1 - \sqrt{3})$$

$$x = -6 + 6\sqrt{3}$$

$$O(\triangle ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot 12 \cdot (-6 + 6\sqrt{3})\sqrt{3} = 108 - 36\sqrt{3}$$



- 55** Zie de figuur hiernaast.

Stel $AP = x$.

In $\triangle APR$ is $PR = \sqrt{2} \cdot AP = x\sqrt{2}$.

Het is een regelmatige achthoek, dus $PQ = PR = x\sqrt{2}$.

$$AB = AP + PQ + BQ = x + x\sqrt{2} + x = 2x + x\sqrt{2}$$

$$2x + x\sqrt{2} = 6$$

$$x(2 + \sqrt{2}) = 6$$

$$x = \frac{6}{2 + \sqrt{2}}$$

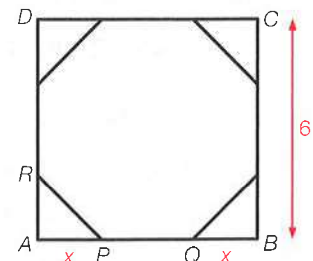
$$x = \frac{6}{2 + \sqrt{2}} \cdot \frac{2 - \sqrt{2}}{2 - \sqrt{2}}$$

$$x = \frac{6(2 - \sqrt{2})}{4 - 2}$$

$$x = 3(2 - \sqrt{2})$$

$$x = 6 - 3\sqrt{2}$$

$$\text{De zijde van de achthoek is } x\sqrt{2} = (6 - 3\sqrt{2})\sqrt{2} = 6\sqrt{2} - 6.$$



Bladzijde 126**56**Stel $AP = x$, dan is ook $BQ = x$.In $\triangle APS$ is $PS = \sqrt{3} \cdot AP = x\sqrt{3}$. $PQRS$ is een vierkant, dus $PQ = PS = x\sqrt{3}$.

$$\left. \begin{aligned} AB &= AP + PQ + BQ = x + x\sqrt{3} + x = 2x + x\sqrt{3} \\ AB &= 8 \end{aligned} \right\} 2x + x\sqrt{3} = 8$$

$$x(2 + \sqrt{3}) = 8$$

$$x = \frac{8}{2 + \sqrt{3}}$$

$$x = \frac{8}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$x = \frac{8(2 - \sqrt{3})}{4 - 3}$$

$$x = 8(2 - \sqrt{3})$$

$$x = 16 - 8\sqrt{3}$$

$$PQ = x\sqrt{3} = (16 - 8\sqrt{3})\sqrt{3} = 16\sqrt{3} - 24$$

57

Zie de figuur hiernaast.

Stel $BP = BQ = CQ = CR = x$.Dan is $AB = x + 1$ en $BC = 2x$.In $\triangle ABC$ is $BC = \sqrt{2} \cdot AB = (x + 1) \cdot \sqrt{2}$.

$$\text{Dus } 2x = (x + 1) \cdot \sqrt{2}$$

$$2x = x\sqrt{2} + \sqrt{2}$$

$$2x - x\sqrt{2} = \sqrt{2}$$

$$x(2 - \sqrt{2}) = \sqrt{2}$$

$$x = \frac{\sqrt{2}}{2 - \sqrt{2}}$$

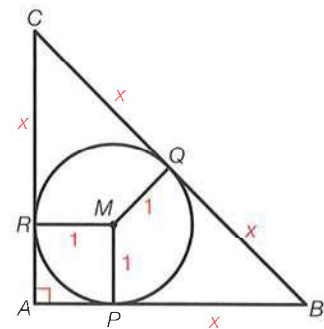
$$x = \frac{\sqrt{2}}{2 - \sqrt{2}} \cdot \frac{2 + \sqrt{2}}{2 + \sqrt{2}}$$

$$x = \frac{\sqrt{2}(2 + \sqrt{2})}{4 - 2}$$

$$x = \frac{2\sqrt{2} + 2}{2}$$

$$x = \sqrt{2} + 1$$

$$\text{De omtrek is } 4x + 2 = 4(\sqrt{2} + 1) + 2 = 6 + 4\sqrt{2}.$$

**58**

Zie de figuur hiernaast.

Het frame is gelijkzijdig, dus $\angle BAC = 60^\circ$.

De benen van hoek A raken de cirkel met middelpunt M en straal r , dus de cirkel met middelpunt M en straal r is ingeschreven cirkel van een driehoek waarvan hoek A een hoekpunt is.

Hieruit volgt dat AM de bissectrice van hoek A is.

$$\text{Dus } \angle BAM = 60^\circ : 2 = 30^\circ.$$

$$\angle ABM = 90^\circ \text{ (stelling raaklijn aan cirkel)}$$

$$\text{Dus in } \triangle ABM \text{ is } AB = \sqrt{3} \cdot BM = r\sqrt{3}.$$

De lengte van de zijde van het frame is

$$r\sqrt{3} + r + 2r + 2r + 2r + r + r\sqrt{3} = 8r + 2r\sqrt{3}.$$

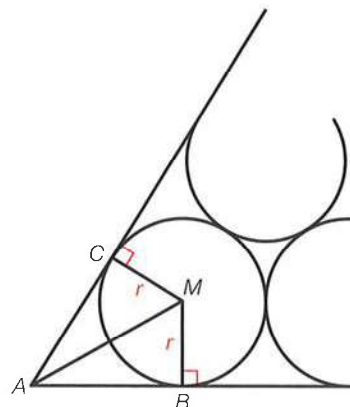
$$\text{Dus } 8r + 2r\sqrt{3} = 30$$

$$r(8 + 2\sqrt{3}) = 30$$

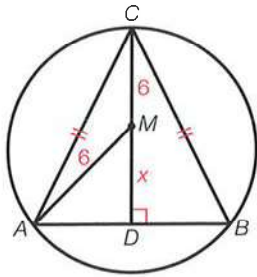
$$r = \frac{30}{8 + 2\sqrt{3}}$$

De diameter van één bal is

$$\frac{30}{4 + \sqrt{3}} = \frac{30}{4 + \sqrt{3}} \cdot \frac{4 - \sqrt{3}}{4 - \sqrt{3}} = \frac{30(4 - \sqrt{3})}{16 - 3} = \frac{30}{13}(4 - \sqrt{3}) = \frac{120}{13} - \frac{30}{13}\sqrt{3} = 9\frac{3}{13} - 2\frac{4}{13}\sqrt{3}.$$



59 **a**



- b** $AB = CD = DM + CM = x + 6$
 $AD = \frac{1}{2}AB = \frac{1}{2}x + 3$
- c** De stelling van Pythagoras in $\triangle ADM$ geeft $AD^2 + DM^2 = AM^2$
 $(\frac{1}{2}x + 3)^2 + x^2 = 6^2$
 $\frac{1}{4}x^2 + 3x + 9 + x^2 = 36$
 $1\frac{1}{4}x^2 + 3x - 27 = 0$
- d** $1\frac{1}{4}x^2 + 3x - 27 = 0$
 $5x^2 + 12x - 108 = 0$
 $D = 12^2 - 4 \cdot 5 \cdot -108 = 2304$, dus $\sqrt{D} = 48$
 $x = \frac{-12 + 48}{10} = 3\frac{3}{5} \vee x = \frac{-12 - 48}{10} = -6$ (kan niet)
 $AB = x + 6 = 9\frac{3}{5}$

Bladzijde 127

60 Zie de figuur hiernaast.
 M is het middelpunt van de cirkel.

Dus $AM = CM = 6$.

Stel $AD = x$, dan is $AB = 2x$, $CD = 3x$ en $DM = 3x - 6$.

De stelling van Pythagoras in $\triangle ADM$ geeft

$$AD^2 + DM^2 = AM^2$$

$$x^2 + (3x - 6)^2 = 6^2$$

$$x^2 + 9x^2 - 36x + 36 = 36$$

$$10x^2 - 36x = 0$$

$$2x(5x - 18) = 0$$

$$2x = 0 \vee 5x = 18$$

$$x = 0 \vee x = 3\frac{3}{5}$$

Dus $AB = 2 \cdot 3\frac{3}{5} = 7\frac{1}{5}$.

61 Zie de figuur hiernaast.

Stel $AP = DP = PQ = BQ = CQ = x$.

$$PR = \frac{13 - x}{2} = 6\frac{1}{2} - \frac{1}{2}x$$

De stelling van Pythagoras in $\triangle APR$ geeft

$$PR^2 + AR^2 = AP^2$$

$$(6\frac{1}{2} - \frac{1}{2}x)^2 + 3^2 = x^2$$

$$42\frac{1}{4} - 6\frac{1}{2}x + \frac{1}{4}x^2 + 9 = x^2$$

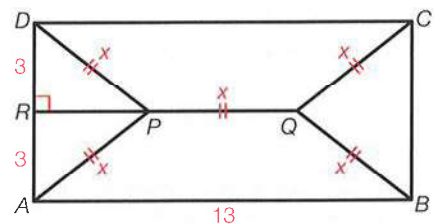
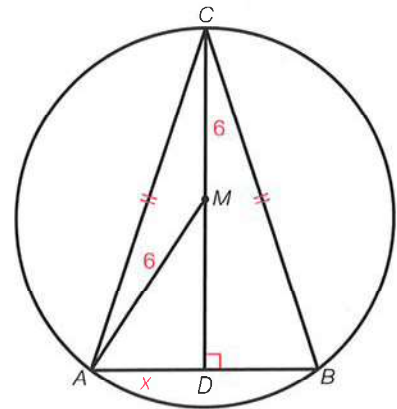
$$\frac{3}{4}x^2 + 6\frac{1}{2}x - 51\frac{1}{4} = 0$$

$$3x^2 + 26x - 205 = 0$$

$$D = 26^2 - 4 \cdot 3 \cdot -205 = 3136, \text{ dus } \sqrt{D} = 56$$

$$x = \frac{-26 + 56}{6} = 5 \vee x = \frac{-26 - 56}{6} = -13\frac{2}{3} \text{ (kan niet)}$$

Dus $AP = 5$.



Bladzijde 128**62** Zie de figuur hiernaast. M is het middelpunt van de cirkel met straal r . $MR \perp BC$ en $MS \perp CD$ (stelling raaklijn aan cirkel).Er geldt $PQ = 11 - 2 - r = 9 - r$ en $QM = 11 - 3 - r = 8 - r$.De stelling van Pythagoras in $\triangle MPQ$ geeft

$$PQ^2 + MQ^2 = MP^2$$

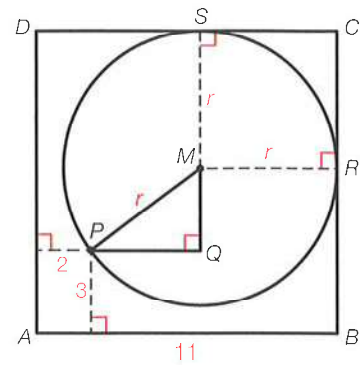
$$(9 - r)^2 + (8 - r)^2 = r^2$$

$$81 - 18r + r^2 + 64 - 16r + r^2 = r^2$$

$$r^2 - 34r + 145 = 0$$

$$(r - 5)(r - 29) = 0$$

$$r = 5 \vee r = 29 \text{ (kan niet)}$$

Dus de straal van de cirkel die door P gaat en de zijden BC en CD raakt is 5.**63** Zie de figuur hiernaast. M is het middelpunt van de cirkels.Stel de breedte van het vlot is x , dan is de breedte van de gracht $2x$.Er geldt $MR = x + 37$ en $MP = 2x + 37$.De stelling van Pythagoras in $\triangle MPR$ geeft

$$PR^2 + MR^2 = MP^2$$

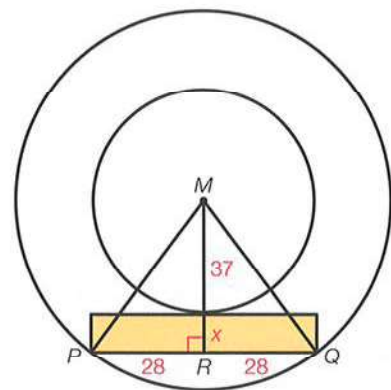
$$28^2 + (x + 37)^2 = (2x + 37)^2$$

$$784 + x^2 + 74x + 1369 = 4x^2 + 148x + 1369$$

$$-3x^2 - 74x + 784 = 0$$

$$D = (-74)^2 - 4 \cdot (-3) \cdot 784 = 14884, \text{ dus } \sqrt{D} = 122$$

$$x = \frac{74 + 122}{-6} = -32\frac{2}{3} \text{ (kan niet)} \vee x = \frac{74 - 122}{-6} = 8$$

Dus de gracht is $2x = 2 \cdot 8 = 16$ meter breed.**64** Zie de figuur hiernaast waarin de grootste cirkel die nog in het overgebleven stuk past is getekend. Deze cirkel heeft middelpunt M en straal r . MB en MC staan loodrecht op de zijden van het vierkant (stelling raaklijn aan cirkel).Er geldt $BM = r$, $AB = 2 - r$ en $AM = r + 1$.De stelling van Pythagoras in $\triangle ABM$ geeft

$$AB^2 + BM^2 = AM^2$$

$$(2 - r)^2 + r^2 = (r + 1)^2$$

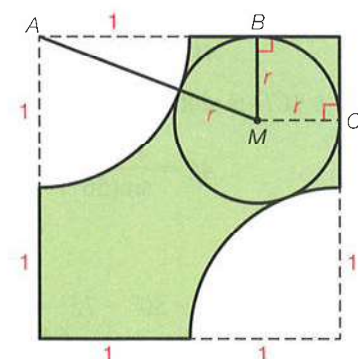
$$4 - 4r + r^2 + r^2 = r^2 + 2r + 1$$

$$r^2 - 6r + 3 = 0$$

$$D = (-6)^2 - 4 \cdot 1 \cdot 3 = 24, \text{ dus } \sqrt{D} = \sqrt{24} = 2\sqrt{6}$$

$$r = \frac{6 + 2\sqrt{6}}{2} \vee r = \frac{6 - 2\sqrt{6}}{2}$$

$$r = 3 + \sqrt{6} \text{ (kan niet)} \vee r = 3 - \sqrt{6}$$

Dus de straal van de grootste cirkel die nog in het overgebleven stuk past is $3 - \sqrt{6}$.

- 65 Zie de figuur hiernaast.
 $O(ABCD) = 1^2 = 1$, dus $O(TUVW) = \frac{1}{2}$, dus $TU = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2}$.

Stel $AT = BU = x$, dan is $AU = x + \frac{1}{2}\sqrt{2}$.

$$O(ABCD) = O(TUVW) + 4 \cdot O(\triangle ABU)$$

$$1 = \frac{1}{2} + 4 \cdot \frac{1}{2} \cdot \left(x + \frac{1}{2}\sqrt{2}\right) \cdot x$$

$$1 = \frac{1}{2} + 2x^2 + x\sqrt{2}$$

$$2x^2 + x\sqrt{2} - \frac{1}{2} = 0$$

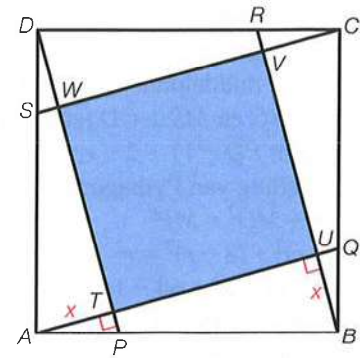
$$D = (\sqrt{2})^2 - 4 \cdot 2 \cdot -\frac{1}{2} = 6$$

$$x = \frac{-\sqrt{2} + \sqrt{6}}{4} = -\frac{1}{4}\sqrt{2} + \frac{1}{4}\sqrt{6} \vee x = \frac{-\sqrt{2} - \sqrt{6}}{4} = -\frac{1}{4}\sqrt{2} - \frac{1}{4}\sqrt{6} \text{ (kan niet)}$$

$$\left. \begin{array}{l} \angle PAT = \angle BAU \\ \angle PTA = \angle BUA \text{ (rechte hoek)} \end{array} \right\} \triangle APT \sim \triangle ABU$$

$$\frac{AP}{AB} = \frac{AT}{AU} \text{ geeft } \frac{AP}{1} = \frac{\frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2}}{\frac{1}{4}\sqrt{6} + \frac{1}{4}\sqrt{2}}$$

$$AP = \frac{\frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2}}{\frac{1}{4}\sqrt{6} + \frac{1}{4}\sqrt{2}} = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} \cdot \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}} = \frac{6 - 2\sqrt{12} + 2}{6 - 2} = \frac{8 - 2 \cdot 2\sqrt{3}}{4} = 2 - \sqrt{3}$$



3.5 De sinusregel en de cosinusregel

Bladzijde 130

- 66 $\angle C = 180^\circ - 48^\circ - 76^\circ = 56^\circ$

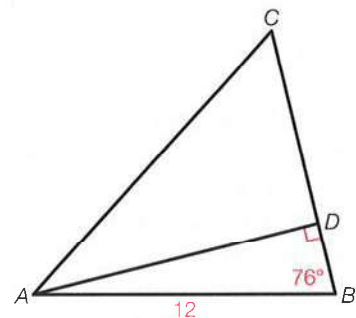
Zie de figuur hiernaast met hoogtelijn AD.

$$\text{In } \triangle ABD \text{ is } \sin(76^\circ) = \frac{AD}{12}$$

$$AD = 12 \sin(76^\circ) = 11,64...$$

$$\text{In } \triangle ACD \text{ is } \sin(56^\circ) = \frac{11,64...}{AC}$$

$$AC = \frac{11,64...}{\sin(56^\circ)} \approx 14,04$$



Bladzijde 131

- 67 a $\angle C = 180^\circ - 50^\circ - 75^\circ = 55^\circ$

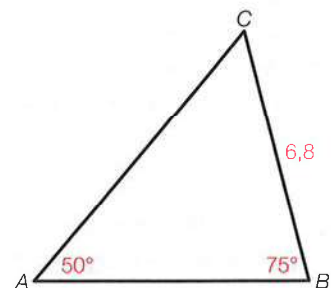
b De sinusregel geeft $\frac{6,8}{\sin(50^\circ)} = \frac{AB}{\sin(55^\circ)}$

$$AB = \frac{6,8 \sin(55^\circ)}{\sin(50^\circ)} \approx 7,3$$

en ook

$$\frac{6,8}{\sin(50^\circ)} = \frac{AC}{\sin(75^\circ)}$$

$$AC = \frac{6,8 \sin(75^\circ)}{\sin(50^\circ)} \approx 8,6$$



- 68 a In $\triangle ACD$ is $\sin(\alpha) = \frac{CD}{b}$, dus $CD = b \sin(\alpha)$.

In $\triangle BCD$ is $\sin(\beta) = \frac{CD}{a}$, dus $CD = a \sin(\beta)$.

- b Uit a volgt $a \sin(\beta) = b \sin(\alpha)$.

Linker- en rechterlid delen door $\sin(\alpha) \cdot \sin(\beta)$ geeft $\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)}$.

c In $\triangle ABE$ is $\sin(\beta) = \frac{AE}{c}$, dus $AE = c \sin(\beta)$.

In $\triangle ACE$ is $\sin(\gamma) = \frac{AE}{b}$, dus $AE = b \sin(\gamma)$.

d Uit c volgt $b \sin(\gamma) = c \sin(\beta)$.

Linker- en rechterlid delen door $\sin(\beta) \cdot \sin(\gamma)$ geeft $\frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$.

$$\text{e } \left. \begin{array}{l} \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} \\ \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)} \end{array} \right\} \text{ dus } \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

69 a In $\triangle ACD$ is $\sin(\angle DAC) = \sin(180^\circ - \alpha) = \frac{CD}{b}$, dus $CD = b \sin(180^\circ - \alpha)$.

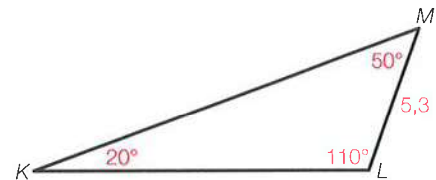
b In $\triangle BCD$ is $\sin(\beta) = \frac{CD}{a}$, dus $CD = a \sin(\beta)$.

c Uit a en b volgt $b \sin(180^\circ - \alpha) = a \sin(\beta)$.

Linker- en rechterlid delen door $\sin(180^\circ - \alpha) \cdot \sin(\beta)$ geeft $\frac{a}{\sin(180^\circ - \alpha)} = \frac{b}{\sin(\beta)}$.

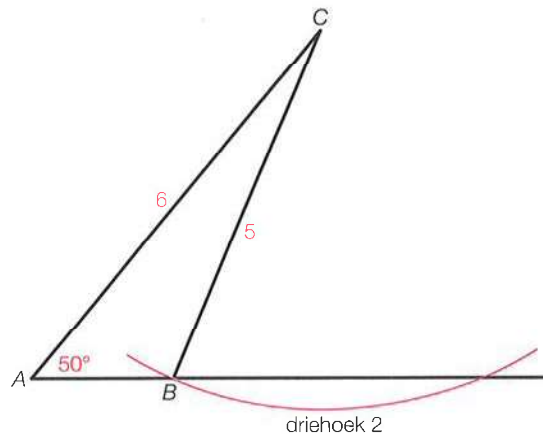
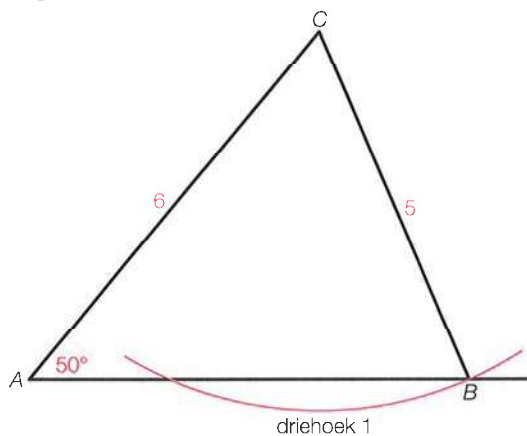
70 $\angle M = 180^\circ - 20^\circ - 110^\circ = 50^\circ$

De sinusregel geeft $\frac{5,3}{\sin(20^\circ)} = \frac{KL}{\sin(50^\circ)}$
 $KL = \frac{5,3 \sin(50^\circ)}{\sin(20^\circ)} \approx 11,9$
 en ook $\frac{5,3}{\sin(20^\circ)} = \frac{KM}{\sin(110^\circ)}$
 $KM = \frac{5,3 \sin(110^\circ)}{\sin(20^\circ)} \approx 14,6$



Bladzijde 132

71 a



b De sinusregel in driehoek 1 geeft $\frac{5}{\sin(50^\circ)} = \frac{6}{\sin(\beta)}$
 $\sin(\beta) = \frac{6 \sin(50^\circ)}{5} = 0,91...$
 $\beta \approx 66,8^\circ$

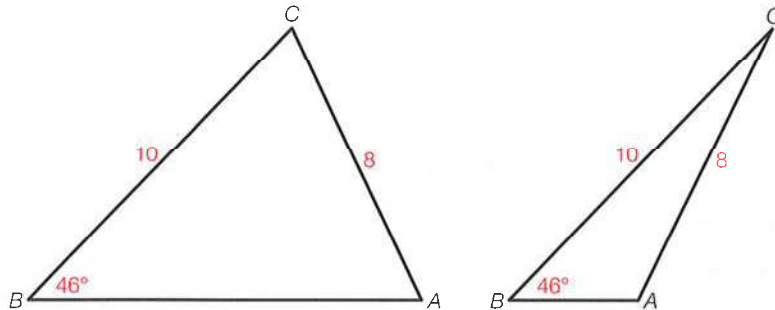
$$\gamma \approx 180^\circ - 50^\circ - 66,8^\circ = 63,2^\circ$$

De sinusregel in driehoek 1 geeft $\frac{5}{\sin(50^\circ)} = \frac{c}{\sin(63,1...^\circ)}$
 $c = \frac{5 \sin(63,1...^\circ)}{\sin(50^\circ)} \approx 5,8$

c $\beta \approx 180^\circ - 66,8^\circ = 113,2^\circ$
 $\gamma \approx 180^\circ - 50^\circ - 113,2^\circ = 16,8^\circ$

De sinusregel in driehoek 2 geeft $\frac{5}{\sin(50^\circ)} = \frac{c}{\sin(16,8\dots^\circ)}$
 $c = \frac{5 \sin(16,8\dots^\circ)}{\sin(50^\circ)} \approx 1,9$

72 a



De sinusregel in de scherphoekige driehoek geeft $\frac{8}{\sin(46^\circ)} = \frac{10}{\sin(\angle A)}$
 $\sin(\angle A) = \frac{10 \sin(46^\circ)}{8}$

$$\angle A \approx 64,0^\circ$$

In de stomphoekige driehoek is $\angle A \approx 180^\circ - 64,0^\circ = 116,0^\circ$.

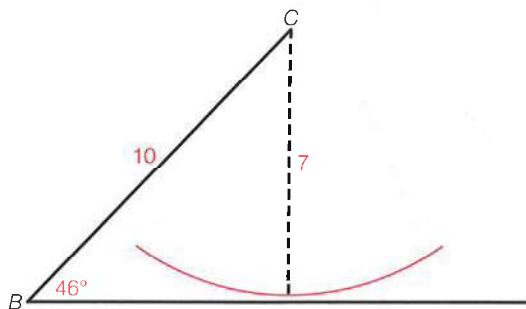
b In de scherphoekige driehoek is $\angle C = 180^\circ - 46^\circ - 64,04\dots^\circ = 69,95\dots^\circ$.

De sinusregel in de scherphoekige driehoek geeft $\frac{8}{\sin(46^\circ)} = \frac{AB}{\sin(69,95\dots^\circ)}$
 $AB = \frac{8 \sin(69,95\dots^\circ)}{\sin(46^\circ)} \approx 10,4$

In de stomphoekige driehoek is $\angle C = 180^\circ - 46^\circ - 115,95\dots^\circ = 18,04\dots^\circ$.

De sinusregel in de stomphoekige driehoek geeft $\frac{8}{\sin(46^\circ)} = \frac{AB}{\sin(18,04\dots^\circ)}$
 $AB = \frac{8 \sin(18,04\dots^\circ)}{\sin(46^\circ)} \approx 3,4$

c



De cirkel met middelpunt C en straal 7 snijdt het andere been van hoek B niet.

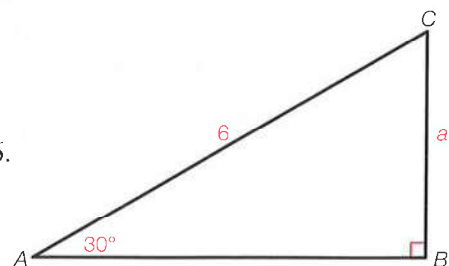
73 a Zie de figuur hiernaast.

$$\sin(30^\circ) = \frac{a}{6}, \text{ dus } a = 6 \sin(30^\circ) = 3.$$

Als er geen driehoek ABC mogelijk is, dan is $a < 3$.

b Als er precies één driehoek ABC mogelijk is, dan is $a = 3 \vee a \geq 6$.

c Als er twee driehoeken ABC mogelijk zijn, dan is $3 < a < 6$.



74 $\angle ADC = 180^\circ - 62^\circ - 48^\circ = 70^\circ$

De sinusregel in $\triangle ACD$ geeft $\frac{6}{\sin(62^\circ)} = \frac{AD}{\sin(48^\circ)}$
 $AD = \frac{6 \sin(48^\circ)}{\sin(62^\circ)} = 5,04...$

en ook

$$\frac{6}{\sin(62^\circ)} = \frac{AC}{\sin(70^\circ)}$$

$$AC = \frac{6 \sin(70^\circ)}{\sin(62^\circ)} = 6,38...$$

$$\angle BDC = 180^\circ - \angle ADC = 180^\circ - 70^\circ = 110^\circ$$

$$\angle DBC = 180^\circ - 110^\circ - 25^\circ = 45^\circ$$

De sinusregel in $\triangle BCD$ geeft $\frac{6}{\sin(45^\circ)} = \frac{BD}{\sin(25^\circ)}$
 $BD = \frac{6 \sin(25^\circ)}{\sin(45^\circ)} = 3,58...$

en ook

$$\frac{6}{\sin(45^\circ)} = \frac{BC}{\sin(110^\circ)}$$

$$BC = \frac{6 \sin(110^\circ)}{\sin(45^\circ)} = 7,97...$$

omtrek $\triangle ABC = AD + BD + BC + AC = 5,04... + 3,58... + 7,97... + 6,38... \approx 23,0$

75 $\angle BCD = 180^\circ - 110^\circ - 40^\circ = 30^\circ$

De sinusregel in $\triangle BCD$ geeft $\frac{CD}{\sin(40^\circ)} = \frac{4}{\sin(30^\circ)}$
 $CD = \frac{4 \sin(40^\circ)}{\sin(30^\circ)} = 5,14...$

$$\angle ACD = \angle ACB - \angle BCD = 50^\circ - 30^\circ = 20^\circ$$

De sinusregel in $\triangle ACD$ geeft $\frac{5,14...}{\sin(\angle CAD)} = \frac{5}{\sin(20^\circ)}$
 $\sin(\angle CAD) = \frac{5,14... \cdot \sin(20^\circ)}{5} = 0,35...$
 $\angle CAD = 20,59...^\circ$

$$\angle ADC = 180^\circ - 20,59...^\circ - 20^\circ = 139,40...^\circ$$

De sinusregel in $\triangle ACD$ geeft $\frac{AC}{\sin(139,40...^\circ)} = \frac{5}{\sin(20^\circ)}$
 $AC = \frac{5 \sin(139,40...^\circ)}{\sin(20^\circ)} \approx 9,51$

Bladzijde 133

76 a De sinusregel geeft $\frac{4}{\sin(\angle A)} = \frac{5}{\sin(\angle B)} = \frac{6}{\sin(\angle C)}$

Bij elke combinatie van twee breuken zijn er twee onbekenden.
 Je kunt de sinusregel dus niet gebruiken.

b De sinusregel geeft $\frac{QR}{\sin(50^\circ)} = \frac{5}{\sin(\angle Q)} = \frac{6}{\sin(\angle R)}$

Bij elke combinatie van twee breuken zijn er twee onbekenden.
 Je kunt de sinusregel dus niet gebruiken.

77 a De stelling van Pythagoras in $\triangle ACD$ geeft $AD^2 + CD^2 = AC^2$
 $x^2 + h^2 = b^2$

b De stelling van Pythagoras in $\triangle BCD$ geeft $BC^2 = BD^2 + CD^2$
 $a^2 = (c - x)^2 + h^2$
 $a^2 = c^2 - 2cx + x^2 + h^2$

- c Vervangen van $x^2 + h^2$ door b^2 in $a^2 = c^2 - 2cx + x^2 + h^2$ geeft $a^2 = c^2 - 2cx + b^2$,
oftewel $a^2 = b^2 + c^2 - 2cx$.
- d In $\triangle ACD$ is $\cos(\alpha) = \frac{x}{b}$, dus $x = b \cos(\alpha)$.
- e Vervangen van x door $b \cos(\alpha)$ in $a^2 = b^2 + c^2 - 2cx$ geeft $a^2 = b^2 + c^2 - 2cb \cos(\alpha)$,
oftewel $a^2 = b^2 + c^2 - 2bc \cos(\alpha)$.

Bladzijde 134

- 78 a De stelling van Pythagoras in $\triangle ACD$ geeft $AD^2 + CD^2 = AC^2$

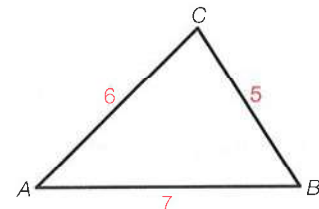
$$x^2 + h^2 = b^2$$

 De stelling van Pythagoras in $\triangle BCD$ geeft $BC^2 = BD^2 + CD^2$

$$a^2 = (c+x)^2 + h^2$$

$$a^2 = c^2 + 2cx + x^2 + h^2$$
- b Vervangen van $x^2 + h^2$ door b^2 in $a^2 = c^2 + 2cx + x^2 + h^2$ geeft $a^2 = c^2 + 2cx + b^2$,
oftewel $a^2 = b^2 + c^2 + 2cx$.
- c In $\triangle ACD$ is $\angle CAD = 180^\circ - \alpha$ en $\cos(\angle CAD) = \frac{AD}{AC}$.
 Dus $\cos(180^\circ - \alpha) = \frac{x}{b}$ oftewel $x = b \cos(180^\circ - \alpha)$.
- d
$$\left. \begin{array}{l} a^2 = b^2 + c^2 + 2cx \\ x = -b \cos(\alpha) \end{array} \right\} a^2 = b^2 + c^2 - 2bc \cos(\alpha)$$

 Dus de cosinusregel geldt ook voor stomphoekige driehoeken.



- 79 a De cosinusregel geeft

$$5^2 = 7^2 + 6^2 - 2 \cdot 7 \cdot 6 \cdot \cos(\angle A)$$

$$25 = 49 + 36 - 84 \cos(\angle A)$$

$$84 \cos(\angle A) = 60$$

$$\cos(\angle A) = \frac{60}{84}$$

$$\angle A = 44,41...^\circ \approx 44,4^\circ$$

 en ook

$$6^2 = 7^2 + 5^2 - 2 \cdot 7 \cdot 5 \cdot \cos(\angle B)$$

$$36 = 49 + 25 - 70 \cos(\angle B)$$

$$70 \cos(\angle B) = 38$$

$$\cos(\angle B) = \frac{38}{70}$$

$$\angle B = 57,12...^\circ \approx 57,1^\circ$$

$$\angle C = 180^\circ - 44,41...^\circ - 57,12...^\circ \approx 78,5^\circ$$
- b De cosinusregel geeft

$$4^2 = 5^2 + 7^2 - 2 \cdot 5 \cdot 7 \cdot \cos(\angle D)$$

$$16 = 25 + 49 - 70 \cos(\angle D)$$

$$70 \cos(\angle D) = 58$$

$$\cos(\angle D) = \frac{58}{70}$$

$$\angle D = 34,04...^\circ \approx 34,0^\circ$$

 en ook

$$7^2 = 5^2 + 4^2 - 2 \cdot 5 \cdot 4 \cdot \cos(\angle E)$$

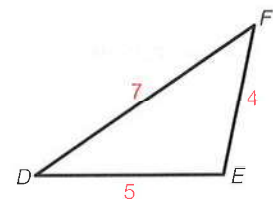
$$49 = 25 + 16 - 40 \cos(\angle E)$$

$$40 \cos(\angle E) = -8$$

$$\cos(\angle E) = \frac{-8}{40}$$

$$\angle E = 101,53...^\circ \approx 101,5^\circ$$

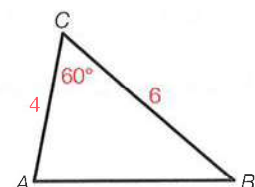
$$\angle F = 180^\circ - 101,53...^\circ - 34,04...^\circ \approx 44,4^\circ$$

**Bladzijde 135**

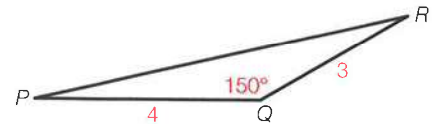
- 80 a De cosinusregel geeft $AB^2 = 4^2 + 6^2 - 2 \cdot 4 \cdot 6 \cdot \cos(60^\circ)$

$$AB^2 = 28$$

$$AB = \sqrt{28} = 2\sqrt{7}$$



- b** De cosinusregel geeft $PR^2 = 4^2 + 3^2 - 2 \cdot 4 \cdot 3 \cdot \cos(150^\circ)$
 $PR^2 = 45,78\dots$
 $PR \approx 6,77$



- 81 a** Zie de figuur hiernaast.

De diagonalen AC en BD snijden elkaar in het punt S met S het midden van AC en van BD .

De cosinusregel in $\triangle ABS$ geeft

$$4,5^2 = 10^2 + 7^2 - 2 \cdot 10 \cdot 7 \cdot \cos(\angle SAB)$$

$$20,25 = 100 + 49 - 140 \cos(\angle SAB)$$

$$140 \cos(\angle SAB) = 128,75$$

$$\cos(\angle SAB) = \frac{128,75}{140}$$

De cosinusregel in $\triangle ABC$ geeft

$$BC^2 = 10^2 + 14^2 - 2 \cdot 10 \cdot 14 \cdot \frac{128,75}{140}$$

$$BC^2 = 38,5$$

$$BC \approx 6,2$$

- b** Zie de figuur hiernaast.

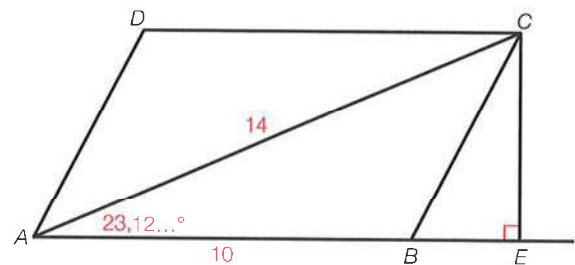
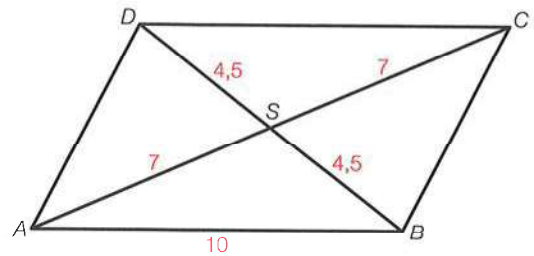
$$\cos(\angle SAB) = \frac{128,75}{140} \text{ geeft } \angle SAB = 23,12\dots^\circ$$

$$\text{In } \triangle ACE \text{ is } \sin(\angle CAE) = \frac{CE}{AC}$$

$$\sin(23,12\dots^\circ) = \frac{CE}{14}$$

$$CE = 14 \sin(23,12\dots^\circ) = 5,49\dots$$

$$O(ABCD) = AB \cdot CE = 10 \cdot 5,49\dots \approx 55,0$$



- 82** De cosinusregel in $\triangle ABD$ geeft $4^2 = 5^2 + 2^2 - 2 \cdot 5 \cdot 2 \cdot \cos(\angle B)$

$$16 = 25 + 4 - 20 \cos(\angle B)$$

$$20 \cos(\angle B) = 13$$

$$\cos(\angle B) = \frac{13}{20}$$

$$\text{De cosinusregel in } \triangle ABC \text{ geeft } AC^2 = 5^2 + (6\frac{1}{2})^2 - 2 \cdot 5 \cdot 6\frac{1}{2} \cdot \frac{13}{20}$$

$$AC^2 = 25$$

$$AC = 5$$

- 83** Zie de figuur hiernaast.

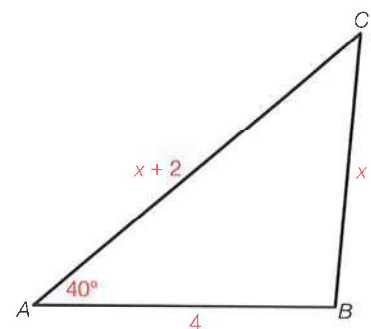
$$\text{De cosinusregel geeft } x^2 = 4^2 + (x+2)^2 - 2 \cdot 4 \cdot (x+2) \cdot \cos(40^\circ)$$

$$\text{oftewel } x^2 = 16 + (x+2)^2 - 8(x+2)\cos(40^\circ).$$

$$\text{Voer in } y_1 = x^2 \text{ en } y_2 = 16 + (x+2)^2 - 8(x+2)\cos(40^\circ).$$

$$\text{De optie snijpunt geeft } x = 3,6381\dots$$

$$\text{Dus } BC \approx 3,638 \text{ en } AC \approx 5,638.$$



Diagnostische toets

Bladzijde 138

- 1** Zie de figuur hiernaast.

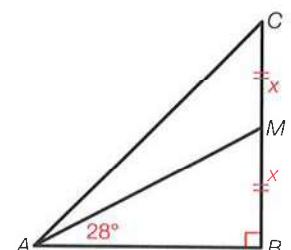
Stel $BM = x$, dan is $BC = 2x$.

$$\text{In } \triangle ABM \text{ is } \tan(28^\circ) = \frac{BM}{AB} = \frac{x}{AB}, \text{ dus } AB = \frac{x}{\tan(28^\circ)}.$$

$$\text{In } \triangle ABC \text{ is } \tan(\angle BAC) = \frac{BC}{AB} = \frac{2x}{\left(\frac{x}{\tan(28^\circ)}\right)} = 2 \tan(28^\circ)$$

$$\angle BAC = 46,76\dots^\circ$$

$$\angle CAM = \angle BAC - \angle BAM = 46,76\dots^\circ - 28^\circ \approx 18,8^\circ$$



- 2 De stelling van Pythagoras in $\triangle ABC$ geeft $AC = \sqrt{15^2 - 12^2} = 9$.

$$CD = AC - AD = 9 - 5 = 4$$

$$\left. \begin{array}{l} \angle ACB = \angle DCE \\ \angle CAB = \angle CDE \text{ (rechte hoek)} \end{array} \right\} \triangle ABC \sim \triangle DEC$$

$$\frac{AB}{DE} \Big| \frac{AC}{CD} \text{ geeft } \frac{12}{DE} \Big| \frac{9}{4}, \text{ dus } DE = \frac{4 \cdot 12}{9} = 5\frac{1}{3}.$$

De stelling van Pythagoras in $\triangle ABD$ geeft $BD = \sqrt{12^2 + 5^2} = 13$.

Stel $BS = x$, dan is $DS = 13 - x$.

$$\left. \begin{array}{l} \angle EAB = \angle AED \text{ (Z-hoeken)} \\ \angle DBA = \angle BDE \text{ (Z-hoeken)} \end{array} \right\} \triangle ABS \sim \triangle EDS \text{ dus } \frac{AB}{DE} \Big| \frac{BS}{DS}$$

$$\text{Dit geeft } \frac{12}{5\frac{1}{3}} \Big| \frac{x}{13-x}$$

$$5\frac{1}{3}x = 12(13-x)$$

$$5\frac{1}{3}x = 156 - 12x$$

$$17\frac{1}{3}x = 156$$

$$x = 9$$

Dus $BS = 9$.

- 3 Zie de figuur hiernaast.

$$\left. \begin{array}{l} \angle APM = \angle BPN \\ \angle MAP = \angle NBP \text{ (rechte hoek)} \end{array} \right\} \triangle AMP \sim \triangle BNP$$

Stel $NP = x$, dan is $MP = x + 8$.

$$\frac{AM}{BN} \Big| \frac{MP}{NP} \text{ geeft } \frac{5}{3} \Big| \frac{x+8}{x}$$

$$5x = 3(x+8)$$

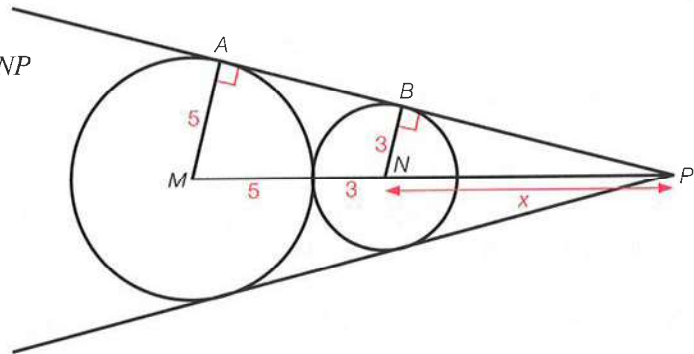
$$5x = 3x + 24$$

$$2x = 24$$

$$x = 12$$

Dus $NP = 12$ en $MP = 12 + 8 = 20$.

In $\triangle AMP$ is $AP = \sqrt{20^2 - 5^2} = \sqrt{375} = 5\sqrt{15}$.



- 4 a Zie de figuur hiernaast.

AC is raaklijn aan de cirkel met raakpunt E , dus

$\angle CEM = 90^\circ$ (stelling raaklijn aan cirkel).

CD is symmetrieas van $\triangle ABC$, dus D is het midden

van AB , CD staat loodrecht op AB , en CD is de

bissectrice van $\angle ACB$ en gaat dus door M .

$$\left. \begin{array}{l} \angle ADC = \angle CEM \text{ (rechte hoek)} \\ \angle ACD = \angle ECM \end{array} \right\} \triangle ACD \sim \triangle MCE \text{ dus } \frac{AC}{CM} \Big| \frac{AD}{EM}$$

De stelling van Pythagoras in $\triangle ACD$ geeft $CD = \sqrt{10^2 - 6^2} = 8$.

Stel $DM = EM = r$, dan is $CM = 8 - r$.

$$\text{Dit geeft } \frac{10}{8-r} \Big| \frac{6}{r}$$

$$10r = 6(8-r)$$

$$10r = 48 - 6r$$

$$16r = 48$$

$$r = 3$$

De straal van de ingeschreven cirkel is 3.

- b Zie de figuur hiernaast.

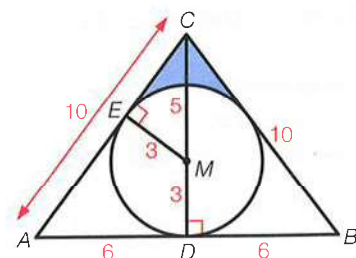
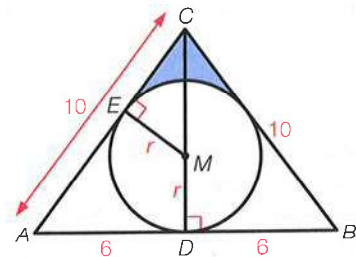
$$CM = CD - DM = 8 - 3 = 5$$

De stelling van Pythagoras in $\triangle CEM$ geeft $CE = \sqrt{5^2 - 3^2} = 4$.

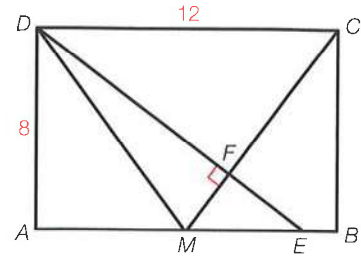
$$O(\triangle CEM) = \frac{1}{2} \cdot CE \cdot EM = \frac{1}{2} \cdot 4 \cdot 3 = 6$$

In $\triangle CEM$ is $\cos(\angle CME) = \frac{3}{5}$, dus $\angle CME = 53,13^\circ$.

De gevraagde oppervlakte is $2 \cdot \left(6 - \frac{53,13^\circ}{360} \cdot \pi \cdot 3^2\right) \approx 3,65$.



- 5 a** De stelling van Pythagoras in $\triangle BCM$ geeft $CM = \sqrt{6^2 + 8^2} = 10$.
Teken DM . Zie de figuur hiernaast.
De zijde $\times$ hoogte-methode in $\triangle CDM$ geeft $CM \cdot DF = CD \cdot AD$
 $10 \cdot DF = 12 \cdot 8$
 $DF = \frac{12 \cdot 8}{10} = 9\frac{3}{5}$



- b** De stelling van Pythagoras in $\triangle CDF$ geeft $CF = \sqrt{12^2 - (9\frac{3}{5})^2} = 7\frac{1}{5}$.
 $FM = CM - CF = 10 - 7\frac{1}{5} = 2\frac{4}{5}$

$$\left. \begin{array}{l} \angle CME = \angle MCD \text{ (Z-hoeken)} \\ \angle DEM = \angle EDC \text{ (Z-hoeken)} \end{array} \right\} \triangle EMF \sim \triangle DCF$$

$$\frac{EM}{CD} \Bigg| \frac{FM}{CF} \text{ geeft } \frac{EM}{12} \Bigg| \frac{2\frac{4}{5}}{7\frac{1}{5}}, \text{ dus } EM = \frac{12 \cdot 2\frac{4}{5}}{7\frac{1}{5}} = 4\frac{2}{3}.$$

Bladzijde 139

- 6 a** $\sqrt{45} - \frac{8}{\sqrt{5}} = 3\sqrt{5} - \frac{8}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = 3\sqrt{5} - \frac{8\sqrt{5}}{5} = 3\sqrt{5} - 1\frac{3}{5}\sqrt{5} = 1\frac{2}{5}\sqrt{5}$
b $\sqrt{18a} + \sqrt{\frac{2}{9}a} = \sqrt{9} \cdot \sqrt{2a} + \sqrt{\frac{1}{9}} \cdot \sqrt{2a} = 3\sqrt{2a} + \frac{1}{3}\sqrt{2a} = 3\frac{1}{3}\sqrt{2a}$
c $\sqrt{12\frac{1}{2}} - \sqrt{4\frac{1}{2}} = \sqrt{\frac{25}{2}} - \sqrt{\frac{9}{2}} = \frac{\sqrt{25}}{\sqrt{2}} - \frac{\sqrt{9}}{\sqrt{2}} = \frac{5}{\sqrt{2}} - \frac{3}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$
d $\frac{3\sqrt{5}}{2 - \sqrt{5}} = \frac{3\sqrt{5}}{2 - \sqrt{5}} \cdot \frac{2 + \sqrt{5}}{2 + \sqrt{5}} = \frac{3\sqrt{5}(2 + \sqrt{5})}{4 - 5} = -3\sqrt{5}(2 + \sqrt{5}) = -6\sqrt{5} - 15$
e $(2a\sqrt{5} - 3)(2a\sqrt{5} + 3) = (2a\sqrt{5})^2 - 3^2 = 20a^2 - 9$
f $\frac{2\sqrt{3}}{\sqrt{6} - \sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{6} - \sqrt{3}} \cdot \frac{\sqrt{6} + \sqrt{3}}{\sqrt{6} + \sqrt{3}} = \frac{2\sqrt{3}(\sqrt{6} + \sqrt{3})}{6 - 3} = \frac{2\sqrt{18} + 6}{3} = \frac{2 \cdot 3\sqrt{2} + 6}{3} = 2\sqrt{2} + 2$

- 7 a** $x\sqrt{3} + 4 = \sqrt{6}$
 $x\sqrt{3} = \sqrt{6} - 4$
 $x = \frac{\sqrt{6}}{\sqrt{3}} - \frac{4}{\sqrt{3}}$
 $x = \sqrt{2} - \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$
 $x = \sqrt{2} - \frac{4\sqrt{3}}{3}$
 $x = \sqrt{2} - 1\frac{1}{3}\sqrt{3}$
b $2x\sqrt{3} + 3x - 12 = 0$
 $x(2\sqrt{3} + 3) = 12$
 $x = \frac{12}{2\sqrt{3} + 3}$
 $x = \frac{12}{2\sqrt{3} + 3} \cdot \frac{2\sqrt{3} - 3}{2\sqrt{3} - 3}$
 $x = \frac{12(2\sqrt{3} - 3)}{12 - 9}$
 $x = 4(2\sqrt{3} - 3)$
 $x = 8\sqrt{3} - 12$

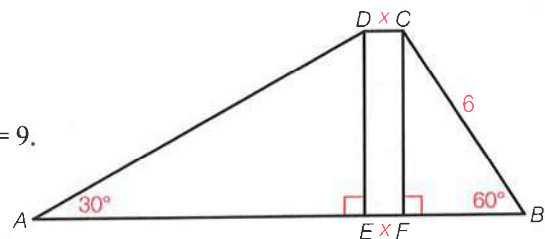
$$\begin{aligned}
 \text{c } 2x\sqrt{2} + x\sqrt{6} &= \sqrt{3} \\
 x(2\sqrt{2} + \sqrt{6}) &= \sqrt{3} \\
 x &= \frac{\sqrt{3}}{2\sqrt{2} + \sqrt{6}} \\
 x &= \frac{\sqrt{3}}{2\sqrt{2} + \sqrt{6}} \cdot \frac{2\sqrt{2} - \sqrt{6}}{2\sqrt{2} - \sqrt{6}} \\
 x &= \frac{\sqrt{3}(2\sqrt{2} - \sqrt{6})}{8 - 6} \\
 x &= \frac{2\sqrt{6} - \sqrt{18}}{2} \\
 x &= \frac{2\sqrt{6} - 3\sqrt{2}}{2} \\
 x &= \sqrt{6} - 1\frac{1}{2}\sqrt{2}
 \end{aligned}$$

8 Zie de figuur hiernaast.

In $\triangle BCF$ is $BF = \frac{1}{2} BC = \frac{1}{2} \cdot 6 = 3$ en $CF = \sqrt{3} \cdot BF = 3\sqrt{3}$.

In $\triangle ADE$ is $DE = CF = 3\sqrt{3}$ en $AE = \sqrt{3} \cdot DE = \sqrt{3} \cdot 3\sqrt{3} = 9$.

Stel $CD = EF = x$.



$$\begin{aligned}
 O(ABCD) &= \frac{1}{2}(AB + CD) \cdot DE = \frac{1}{2}(9 + x + 3 + x) \cdot 3\sqrt{3} = (x + 6) \cdot 3\sqrt{3} \\
 O(ABCD) &= 36
 \end{aligned}$$

$$(x + 6) \cdot 3\sqrt{3} = 36$$

$$x + 6 = \frac{36}{3\sqrt{3}}$$

$$x = \frac{36}{3\sqrt{3}} - 6$$

$$x = \frac{36}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} - 6$$

$$x = \frac{36\sqrt{3}}{9} - 6$$

$$x = 4\sqrt{3} - 6$$

$$\text{Dus } AB = 9 + 4\sqrt{3} - 6 + 3 = 6 + 4\sqrt{3}.$$

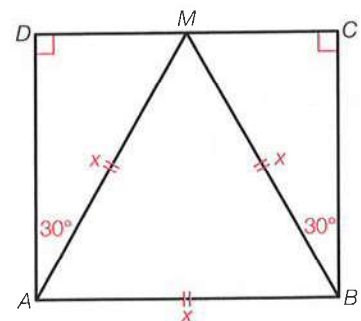
9 $\triangle ABM$ is gelijkzijdig, dus $\angle MAB = \angle MBA = \angle AMB = 60^\circ$.
 $\angle DAM = 90^\circ - \angle MAB = 90^\circ - 60^\circ = 30^\circ$ en $\angle D = 90^\circ$
 Stel $AB = BM = AM = x$.

Dan geldt in $\triangle ADM$ dat $DM = \frac{x}{2} = \frac{1}{2}x$ en $AD = \frac{1}{2}x\sqrt{3}$.

$$\begin{aligned}
 \text{omtrek } ABCD &= AB + BC + CD + AD \\
 &= 2 \cdot x + 2 \cdot \frac{1}{2}x\sqrt{3} = 2x + x\sqrt{3} \\
 \text{omtrek } ABCD &= 10
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{aligned} 2x + x\sqrt{3} &= 10 \\ x(2 + \sqrt{3}) &= 10 \\ x &= \frac{10}{2 + \sqrt{3}} \end{aligned}$$

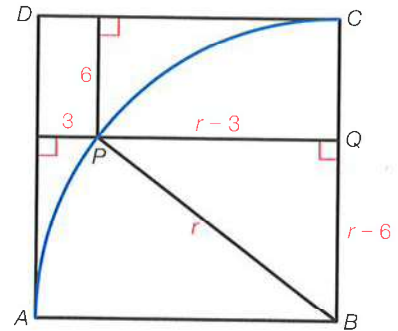
Dus de omtrek van driehoek ABM is

$$3 \cdot \frac{10}{2 + \sqrt{3}} = \frac{30}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{30(2 - \sqrt{3})}{4 - 3} = 60 - 30\sqrt{3}.$$

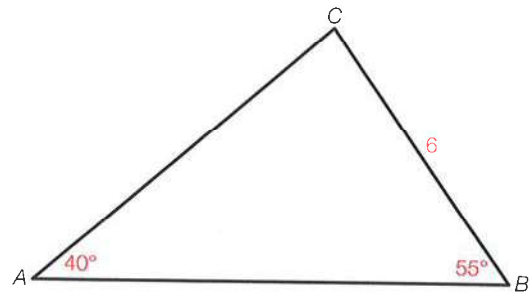


- 10** Zie de figuur hiernaast.
 Stel de straal van de cirkel r .
 Dan is $BP = r$, $BQ = r - 6$ en $PQ = r - 3$.
 De stelling van Pythagoras in $\triangle BPQ$ geeft
 $PQ^2 + BQ^2 = BP^2$
 $(r - 3)^2 + (r - 6)^2 = r^2$
 $r^2 - 6r + 9 + r^2 - 12r + 36 = r^2$
 $r^2 - 18r + 45 = 0$
 $(r - 3)(r - 15) = 0$
 $r = 3 \vee r = 15$
 $r = 3$ kan niet.

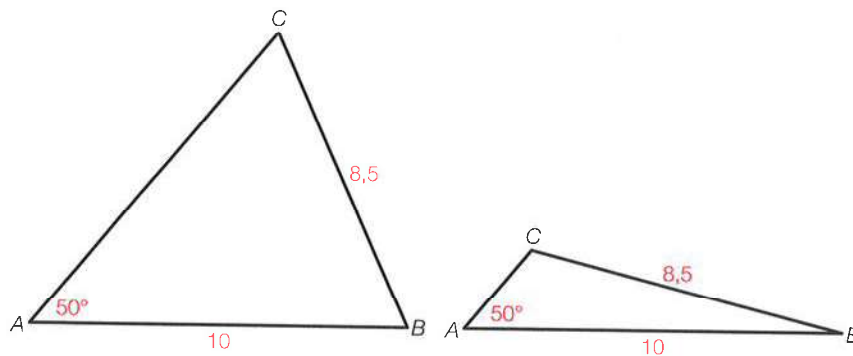
De oppervlakte van het vierkant is $AB \cdot BC = r \cdot r = 15 \cdot 15 = 225$.



- 11** Zie de figuur hiernaast.
 $\angle C = 180^\circ - 40^\circ - 55^\circ = 85^\circ$
 De sinusregel geeft $\frac{6}{\sin(40^\circ)} = \frac{AC}{\sin(55^\circ)}$
 $AC = \frac{6 \sin(55^\circ)}{\sin(40^\circ)} = 7,64\dots$
 $O(\triangle ABC) = \frac{1}{2} \cdot AC \cdot BC \cdot \sin(\angle C)$
 $= \frac{1}{2} \cdot 7,64\dots \cdot 6 \cdot \sin(85^\circ) \approx 22,85$



12 a



De sinusregel in de scherphoekige driehoek geeft $\frac{8,5}{\sin(50^\circ)} = \frac{10}{\sin(\angle C)}$
 $\sin(\angle C) = \frac{10 \sin(50^\circ)}{8,5}$
 $\angle C \approx 64,3^\circ$

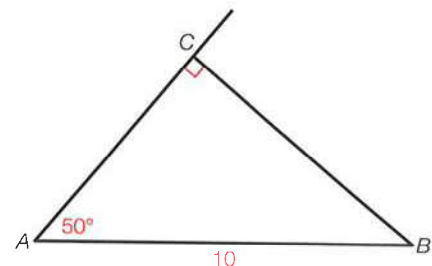
Dus in de scherphoekige driehoek is $\angle B \approx 180^\circ - 50^\circ - 64,3^\circ = 65,7^\circ$.

In de stomphoekige driehoek is $\angle C \approx 180^\circ - 64,3^\circ = 115,7^\circ$ en
 $\angle B \approx 180^\circ - 50^\circ - 115,7^\circ = 14,3^\circ$.

b Zie de figuur hiernaast.

$$\sin(50^\circ) = \frac{BC}{10}, \text{ dus } BC = 10 \sin(50^\circ) = 7,660\dots$$

Als er precies één driehoek ABC mogelijk is, dan is
 $BC \approx 7,66 \vee BC \geq 10$.

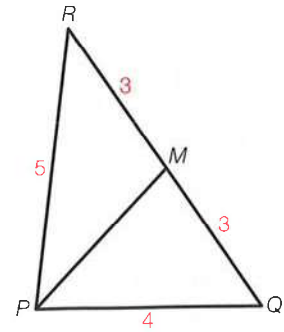


13 Zie de figuur hiernaast.

De cosinusregel in $\triangle PQR$ geeft $4^2 = 5^2 + 6^2 - 2 \cdot 5 \cdot 6 \cdot \cos(\angle R)$
 $16 = 25 + 36 - 60 \cos(\angle R)$
 $60 \cos(\angle R) = 45$
 $\cos(\angle R) = \frac{3}{4}$

De cosinusregel in $\triangle MPR$ geeft $PM^2 = 5^2 + 3^2 - 2 \cdot 5 \cdot 3 \cdot \frac{3}{4}$
 $PM^2 = 11\frac{1}{2}$

De cosinusregel in $\triangle MPR$ geeft $5^2 = 11\frac{1}{2} + 3^2 - 2 \cdot \sqrt{11\frac{1}{2}} \cdot 3 \cdot \cos(\angle PMR)$
 $6\sqrt{11\frac{1}{2}} \cdot \cos(\angle PMR) = -4\frac{1}{2}$
 $\cos(\angle PMR) = \frac{-4\frac{1}{2}}{6\sqrt{11\frac{1}{2}}}$
 $\angle PMR \approx 102,8^\circ$



14 a Zie de figuur hiernaast.

De cosinusregel in $\triangle ABD$ geeft
 $10^2 = 8^2 + 15^2 - 2 \cdot 8 \cdot 15 \cdot \cos(\angle BAD)$
 $100 = 64 + 225 - 240 \cos(\angle BAD)$
 $240 \cos(\angle BAD) = 189$
 $\cos(\angle BAD) = \frac{189}{240}$
 $\angle BAD = 38,04...^\circ$

In $\triangle ADE$ is $\sin(38,04...^\circ) = \frac{DE}{8}$, dus $DE = 8 \sin(38,04...^\circ) = 4,93...$

$O(ABCD) = AB \cdot DE = 15 \cdot 4,93... \approx 73,96$

b $CF = DE = 4,93...$

De stelling van Pythagoras in $\triangle BCF$ geeft $BF = \sqrt{8^2 - 4,93...^2} = 6,3$.

De stelling van Pythagoras in $\triangle ACF$ geeft $AC = \sqrt{(15 + 6,3)^2 + 4,93...^2} \approx 21,9$.

